Laboratory of Adaptive Control

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E1 Introduction to Matlab-Simulink environment

The exercise objective is to introduce (or recall) useful functions of Matlab-Simulink environment through basic analysis of an exemplary mechanical plant in the time and frequency domains.

The plant is depicted in Fig. 1, where $f(t)$ is an input force, m is a cart mass, b is a damping coefficient, and c is a spring stiffness constant. The cart position $x(t)$ (in [m]) can change from its initial (rest) condition $x(0) = 0$ as denoted in Fig. 1 (note: for the rest condition holds: $\dot{x}(0) = v(0) \equiv 0$). For simplicity, we assume that the cart wheels can roll without any energy dissipation.

For the purpose of the exercise let us take: $m = 0.5\,\text{kg}$, $b = 0.8\,\text{N·s/m}$, and $c = 2\,\text{N/m}$.

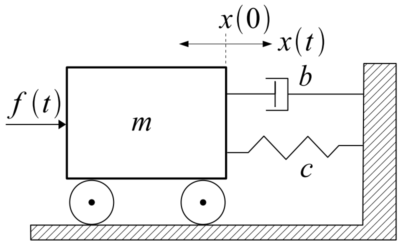

Figure 1: Physical model of a simple mechanical system

1 Time-domain analysis of the plant dynamics

1.1 Using the d'Alembert principle, write differential equation which can model the mechanical system from Fig. 1.

1.2 Treating the force $f(t)$ as a control input, that is $u(t) \triangleq f(t)$, derive the transfer function

$$G(s) \triangleq \frac{X(s)}{U(s)} \tag{1}$$

for zero initial conditions, that is for: $x(0) = 0$, $v(0) = 0$. Write the transfer function in the canonical form of the second-order oscillatory dynamics.

1.3 Define the derived transfer function in Matlab using the `tf` function for values of plant parameters proposed above. Upon a form of the transfer function determine values of dc-gain K, damping factor ζ, and natural frequency ω_n of the plant.

1.4 Compute and plot the step response of $G(s)$ using the `step` function. Utilizing the tools of a figure window in Matlab read the steady-state value $x_{ss} = x(\infty)$, the relative overshoot $\kappa_{\%}$, and settling time T_s of the step response (for the latter use the 2% vicinity of the steady-state value).

1.5 Using the `Integrator` blocks in the Simulink environment build an analog scheme of the plant dynamics upon the differential equation derived in point 1.1 Obtain a step response of the plant in the Simulink environment – plot $x(t)$ and $v(t)$ in the `Scope` block and write the step response samples together with time instants and input signal samples to matrix `PlantData`. Plot the signals $x(t)$ and $u(t)$ in the common figure in Matlab (use function `plot`) taking the samples from matrix `PlantData`.

1.6 Define the transfer function $G(s)$ in Simulink environment using the `Transfer Fcn` block. Again, obtain a step response of the plant in the Simulink environment and plot it in the `Scope` block. Find the qualitative differences between the two forms of the plant model (i.e. the analog scheme and the transfer function) implemented in Simulink.

2 Frequency-domain analysis of the plant dynamics

2.1 In Simulink environment obtain a response of the plant to sinusoidal input signal

$$u(t) \triangleq \sin(\omega t) \tag{2}$$

for $\omega = 1.51\,\text{rad/s}$. Plot the signals $u(t)$ and $x(t)$ in the same `Scope` block. Qualitatively compare the input and the steady output. What can be seen?

2.2 Employing function `bode` in Matlab, plot the frequency characteristics of transfer function $G(j\omega)$. Upon the Bode plot, read values of the amplitude and phase shift of a sinusoidal steady response $x(t)$ for frequency $\omega = 1.51\,\text{rad/s}$. Quantitatively compare the readings with the plots obtained in point 2.1.

2.3 In Simulink environment obtain a response of the plant to signal (2) for $\omega = 11.6\,\text{rad/s}$.

2.4 Upon the Bode plot of transfer function $G(j\omega)$, read values of the amplitude and phase shift of a sinusoidal steady response $x(t)$ for frequency $\omega = 11.6\,\text{rad/s}$. Quantitatively compare the readings with the plots obtained in point 2.3. What can be said about general properties of the analyzed plant dynamics?

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