Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Off-track reduction control for roundabout negotiation with multi-steering N-trailers

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Abstract: Negotiating a roundabout with long articulated vehicles without any specialized control strategy may be dangerous or even impossible due to an excessively large off-track caused by the towed trailers. The paper provides a generic kinematic steering strategy for this type of maneuver executed with articulated vehicles towing an arbitrary number of trailers equipped with steerable wheels, and attached through an arbitrary hitching type (on- or off-axle one). The control strategy requires a feedback from the *internal* (easily measurable) configuration variables of a vehicle, guarantees zero off-tracking in steady motion conditions, and improves a transient response of a vehicle chain by delaying the reference steering signals with the dynamically (on-line) scheduled time-delays. A resultant control performance is illustrated by simulation results.

Keywords: off-track reduction, multi-steering N-trailer, kinematics, roundabout negotiation

1. INTRODUCTION

Recent trends in the modern freight and public transportation lead to the increasing use of the so-called Large Capacity Vehicles (LCVs), Leduc (2009); Odhams et al. (2009). Such vehicles are usually formed by a chain of trailers making the LCVs long articulated structures. Maneuvering with long vehicles is difficult and dangerous, particularly along narrow passages like roundabouts or urban streets. One of the most common difficulties comes from the off-tracking behaviour of the trailers, resulting in the cut-in motion, when a vehicle corners with a large motion curvature of a tractor unit. Therefore, the problem of reducing the off-track for long articulated vehicles for most common maneuvers is an important practical issue which requires effective solutions concerning both a mechanical design and an active control of LCVs.

A problem of the off-track reduction for articulated vehicles has been largely addressed so far both in the area of robotics, see e.g. Orosco-Guerrero et al. (2002); Ritzen et al. (2016), and in the transportation vehicles literature, Jujnovich and Cebon (2002); Oldhams et al. (2010); Jujnovich and Cebon (2013); Ding et al. (2013); Kharrazi et al. (2013); Wagner et al. (2013); Kural et al. (2017); Miao and Cebon (2018). However, most of the proposed solutions are either developed for a single- or two-trailer constructions, or require substantial changes in a vehicle construction – see Nakamura et al. (2000); Manesis et al. (2003). Hence, the problem is still open in the context of finding more generic control methodologies. In particular, we are searching for a generic solution to a common problem of maneuvering along a roundabout which could be applicable to arbitrary number of trailers interconnected with any of the conventional hitching, that is, with the on-axle or off-axle joints (the latter with positive or negative hitching offsets), see Michałek (2013). In contrast to the control methods which are quite general but are devoted to vehicles with non-steerable wheels of trailers, Altafini (2003); Michałek (2015), in this paper one proposes a new generic control strategy for the vehicles equipped with trailers having actively steerable wheels. The contribution comes from the following ingredients:

- a generic N-trailer kinematic chain of articulated vehicles with steerable trailers' wheels is considered, admitting an *arbitrary* number of trailers and *arbitrary* interconnection types (on-axle and off-axle ones) between the vehicle bodies,
- the proposed control strategy leads to zero steady off-tracking along a roundabout for all the vehicle bodies,
- an analytical solution for steady values of joint angles and steering angles leading to the zero steady off-tracking is derived as a function of a motion curvature of a tractor,
- a relatively simple tuning strategy for the proposed control law has been introduced,
- practical implementation of the proposed controller requires a feedback only from the *internal* (easily measurable) configuration variables of a vehicle.

Performance of the steering control system proposed in the paper has been validated by numerical simulations for a 3-trailer vehicle of nonminimum-phase kinematics.

Notation. For compactness, hereafter we will use short notation: $s\delta \equiv \sin\delta$, $c\delta \equiv \cos\delta$.

2. KINEMATICS OF MULTI-STEERING N-TRAILERS AND A CONTROL PROBLEM

2.1 Vehicle kinematics

A single-track kinematic scheme of the considered multi-steering vehicle is presented in Fig. 1, where the vehicle bodies are numbered by $i = 0, 1, \ldots, N$, starting from the tractor segment and finishing on the last trailer.

The tractor is a car-like prime-mover with a steerable

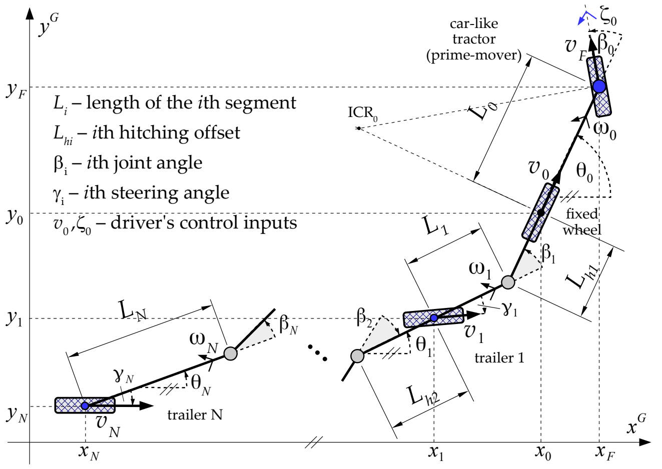

Fig. 1. A single-track kinematic scheme of the considered N-trailer vehicle with steerable wheels of trailers.

front wheel and a fixed rear wheel. Every vehicle segment is equipped with an effective (steerable) wheel and is characterized by two kinematic parameters: the segment length $L_i > 0$ and the hitching offset $L_{hi} \in \mathbb{R}$. If $L_{hi} = 0$ one says about the on-axle hitching, while if $L_{hi} \neq 0$ about the off-axle hitching. The hitching offset $L_{hi} > 0$ if the hitching point is located behind the wheel axle of a preceding vehicle segment, whereas $L_{hi} < 0$ in the opposite case. Assuming no skid-slip motion conditions for all the vehicle wheels, a vehicle configuration can be uniquely represented by the $(S + N + 4)-$dimensional vector

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{\gamma} \\ \beta_0 \\ \boldsymbol{\beta} \\ \boldsymbol{q}_0 \end{bmatrix} \in \mathcal{Q} = [-\breve{\gamma}_s, \breve{\gamma}_s]^S \times [-\breve{\beta}_0, \breve{\beta}_0] \times \mathbb{T}^N \times \mathbb{S}^1 \times \mathbb{R}^2, \quad (1)$$

where $0 < \breve{\gamma}_s, \breve{\beta}_0 < \pi/2$, with indexes $s \in \mathcal{I}_s \subseteq \{1, \ldots, N\}$, determine admissible ranges for the steering variables, $\boldsymbol{\gamma} = [\gamma_{s_1} \ \ldots \ \gamma_{s_S}]^\top$, $s_1, s_S \in \mathcal{I}_s$, is a vector of steering angles of trailer wheels, β_0 is a steering angle of a tractor, $\boldsymbol{\beta} = [\beta_1 \ \ldots \ \beta_N]^\top$ is a vector of joint angles, while $\boldsymbol{q}_0 = [\theta_0 \ x_0 \ y_0]^\top \in \mathbb{S}^1 \times \mathbb{R}^2$ is a body-posture of a prime-mover expressed in a global frame (see Fig. 1). If all the trailers actively use their steerable wheels then $S = N$; in the case when some wheels are not steered then $S < N$ and the angles γ_j for those non-steerable wheels ($j \notin \mathcal{I}_s$) are removed from the configuration vector. Note that $\gamma_0 \equiv 0$ by assumption of the fixed rear wheel of a tractor. The subset $\{\boldsymbol{\gamma}, \beta_0, \boldsymbol{\beta}\}$ of configuration (1) will be called the set of *internal* variables.

Let us denote by ω_i an angular (pseudo-) velocity of the ith vehicle segment (body) and by v_i a longitudinal (pseudo-) velocity of a mid-point (x_i, y_i) of the effective wheel belonging to the ith vehicle segment as presented in Fig. 1. Upon the kinematic analysis presented in detail by Michałek (2019), one can show satisfaction of the following relations:

$$\boldsymbol{u}_i = \underbrace{\begin{bmatrix} -\frac{L_{hi}}{L_i}\frac{\mathrm{c}(\beta_i-\gamma_i)}{\mathrm{c}\gamma_i} & \frac{\mathrm{s}(\beta_i-\gamma_i+\gamma_{i-1})}{L_i\,\mathrm{c}\gamma_i} \\ L_{hi}\frac{\mathrm{s}\beta_i}{\mathrm{c}\gamma_i} & \frac{\mathrm{c}(\beta_i+\gamma_{i-1})}{\mathrm{c}\gamma_i} \end{bmatrix}}_{\boldsymbol{J}_i(\beta_i,\gamma_i,\gamma_{i-1})} \boldsymbol{u}_{i-1}, \quad (2)$$

$$\delta_i = \beta_i - \arctan\left(\frac{L_{hi}\kappa_{i-1}}{\mathrm{c}\gamma_{i-1}} - \tan\gamma_{i-1}\right), \quad (3)$$

$$\kappa_i = \frac{\mathrm{s}(\delta_i - \gamma_i)}{L_i\,\mathrm{c}\delta_i}, \qquad \kappa_0 = \frac{1}{L_0}\tan\beta_0, \quad (4)$$

where (2) is a transformation of velocities $\boldsymbol{u}_j = [\omega_j \ v_j]^\top$ between any two neighbouring segments, δ_i is a virtual steering angle determined for the ith joint, $\kappa_i \triangleq \omega_i/v_i$ is a signed motion curvature of the ith segment, while $\kappa_0 = \kappa_0(\beta_0)$ denotes a signed motion curvature of a prime-mover.

Let us introduce a kinematic control input

$$\boldsymbol{u} = \begin{bmatrix} \boldsymbol{\zeta} \\ \boldsymbol{u}_0 \end{bmatrix} \in \mathbb{R}^{S+2}, \quad \boldsymbol{u}_0 = \begin{bmatrix} \zeta_0 \\ v_0 \end{bmatrix}, \ \boldsymbol{\zeta} = [\zeta_{s_1} \ \ldots \ \zeta_{s_S}]^\top, \quad (5)$$

where ζ_0 is a steering rate of the prime-mover's front wheel, and ζ_{s_j}, $s_j \in \mathcal{I}_s$, are the steering rates of the trailers' steering wheels. Hereafter, we assume that the input $\boldsymbol{u}_0$ is in a disposal of a human driver, whereas $\boldsymbol{\zeta}$ is a control input available for the off-track reduction purposes. Having defined the configuration variables and the control inputs, one can derive a kinematic model of a multi-steering N-trailer in the form of a driftless dynamical system represented by Eq. (6), see Michałek (2019) for more details.

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{\gamma}} \\ \dot{\beta}_0 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \\ \vdots \\ \dot{\beta}_N \\ \dot{\boldsymbol{q}}_0 \end{bmatrix}}_{\dot{\boldsymbol{q}}} = \underbrace{\begin{bmatrix} \boldsymbol{I}_{S\times S} & \boldsymbol{0}_{S\times 1} & \boldsymbol{0}_{S\times 1} \\ \boldsymbol{0}_{1\times S} & 1 & 0 \\ \boldsymbol{0}_{1\times S} & 0 & \boldsymbol{c}^\top \boldsymbol{\Gamma}_1(\beta_1,\gamma_1,\gamma_0)\begin{bmatrix} \kappa_0(\beta_0) \\ 1 \end{bmatrix} \\ \boldsymbol{0}_{1\times S} & 0 & \boldsymbol{c}^\top \boldsymbol{\Gamma}_2(\beta_2,\gamma_2,\gamma_1)\boldsymbol{J}_1(\beta_1,\gamma_1,\gamma_0)\begin{bmatrix} \kappa_0(\beta_0) \\ 1 \end{bmatrix} \\ \vdots & \vdots & \vdots \\ \boldsymbol{0}_{1\times S} & 0 & \boldsymbol{c}^\top \boldsymbol{\Gamma}_N(\beta_N,\gamma_N,\gamma_{N-1})\prod_{j=N-1}^{1}\boldsymbol{J}_j(\beta_j,\gamma_j,\gamma_{j-1})\begin{bmatrix} \kappa_0(\beta_0) \\ 1 \end{bmatrix} \\ \boldsymbol{0}_{3\times S} & 0 & \boldsymbol{G}(\theta_0)\begin{bmatrix} \kappa_0(\beta_0) \\ 1 \end{bmatrix} \end{bmatrix}}_{\boldsymbol{S}(\boldsymbol{q})} \underbrace{\begin{bmatrix} \boldsymbol{\zeta} \\ \boldsymbol{u}_0 \end{bmatrix}}_{\boldsymbol{u}}, \quad \begin{array}{l} \boldsymbol{S}(\boldsymbol{q}) \in \mathbb{R}^{(S+N+4)\times(S+2)}, \\ \boldsymbol{u} \in \mathbb{R}^{S+2}, \\ \boldsymbol{\Gamma}_j(\beta_j,\gamma_j,\gamma_{j-1}) = \boldsymbol{I} - \boldsymbol{J}_j(\beta_j,\gamma_j,\gamma_{j-1}), \\ \boldsymbol{G}(\theta_0) = \begin{bmatrix} 1 & 0 \\ 0 & \mathrm{c}\theta_0 \\ 0 & \mathrm{s}\theta_0 \end{bmatrix}, \\ \boldsymbol{c}^\top = [1 \ 0] \end{array} \quad (6)$$

Let us complement the state equation (6) with an output equation

$$\boldsymbol{y} \triangleq \begin{bmatrix} \boldsymbol{\gamma} \\ \beta_0 \\ \boldsymbol{\beta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_1 \\ \boldsymbol{C}_2 \\ \boldsymbol{C}_3 \end{bmatrix} \boldsymbol{q} = \begin{bmatrix} \boldsymbol{I}_{S\times S} & \boldsymbol{0}_{S\times 1} & \boldsymbol{0}_{S\times N} & \boldsymbol{0}_{S\times 3} \\ \boldsymbol{0}_{1\times S} & 1 & \boldsymbol{0}_{1\times N} & \boldsymbol{0}_{1\times 3} \\ \boldsymbol{0}_{N\times S} & \boldsymbol{0}_{N\times 1} & \boldsymbol{I}_{N\times N} & \boldsymbol{0}_{N\times 3} \end{bmatrix} \boldsymbol{q}, \quad (7)$$

according to which only the internal variables are measurable and available for feedback control design. This important limitation is practically justified by a relative simplicity of precise measuring the internal angular variables by using sensory systems accessible on a vehicle's board (in contrast to the posture variables, $\boldsymbol{q}_0$, which would require an advanced external localization system).

2.2 Control problem formulation

Before stating the control problem under consideration let us introduce four basic performance measures (cf. Jujnovich and Cebon (2002, 2013)).

- **Steady Swept-Path Width** (SSPW): a width of a path swept by the whole articulated vehicle (taking into account vehicle body dimensions) when traversing a roundabout in steady motion conditions.
- **Steady Off-Track** (SOT): a maximal radial distance between the paths drawn by points (x_i, y_i) of trailers and a path drawn by point (x_0, y_0) of a prime-mover (see Fig. 1) when traversing a roundabout in steady motion conditions.
- **Maximal Entrance Swing** (MEnS): a maximal lateral deviation between the paths drawn by points (x_i, y_i) of trailers and a path drawn by point (x_0, y_0) of a prime-mover when entering a roundabout.
- **Maximal Exit Swing** (MExS): a maximal lateral deviation between the paths drawn by points (x_i, y_i) of trailers and a steady path drawn by point (x_0, y_0) of a prime-mover determined when it is lined-up with all the vehicle's segments after exiting a roundabout.

The first two measures reflect a steady-motion performance, while the latter two allow judging a transient performance. In contrast to the conventional approach, the above measures have been intentionally defined with respect to the characteristic points (x_j, y_j), $j \in \{0, \ldots, N\}$, of a vehicle kinematic chain instead of the outer points of vehicle bodies. In this way, it is possible to propose a generic control strategy which will depend only on the kinematic parameters L_i and L_{hi}, which uniquely characterize kinematics of any N-trailer.

Problem 1. The control problem under consideration is to find a steering control law $\boldsymbol{\zeta} = \boldsymbol{\zeta}(\boldsymbol{\gamma}, \beta_0, \boldsymbol{\beta}, \boldsymbol{u}_0)$ which, when applied into kinematics (6), guarantees the zero SOT and sufficiently small SSPW, MEnS, and MExS.

3. STEADY ANGULAR CONFIGURATION FOR CIRCULAR MOTION CONDITIONS

In order to guarantee the zero SOT imposed by Problem 1, we are going to derive formulas for steady joint angles $\bar{\beta}_i$ and steady steering angles $\bar{\gamma}_i$, $i \in \{1, \ldots, N\}$, for a multi-steering N-trailer in the steady circular motion conditions. To this aim, let us consider the case presented in Fig. 2. A finite signed radius R_0 of a steady circle is determined by a steady steering angle $\bar{\beta}_0 \neq 0$ (forced by a driver), namely

$$R_0(\bar{\beta}_0) = L_0 / \tan \bar{\beta}_0. \tag{8}$$

Let us consider behaviour of the ith vehicle segment when all the preceding segments are already in the steady circular motion conditions. In this case $R_{i-1} = R_{i-2} = \ldots = R_0(\bar{\beta}_0)$, where R_j denotes a signed steady curvature radius for the jth segment. Upon elementary geometry provided in Fig. 2, one can derive the following relation for a signed steady motion curvature radius $R_i^*(R_0(\bar{\beta}_0))$ of the ith segment with a non-steerable wheel (that is, when $\gamma_i \equiv 0$):

$$\begin{aligned} R_i^* &= \mathrm{sign}(R_0)\sqrt{R_{i-1}^2 + L_{hi}^2 - L_i^2 + 2\,|R_{i-1}|\,L_{hi}\,\mathrm{s}|\bar{\gamma}_{i-1}|} \\ &= \mathrm{sign}(R_0)\sqrt{R_0^2 + L_{hi}^2 - L_i^2 + 2\,|R_0|\,L_{hi}\,\mathrm{s}|\bar{\gamma}_{i-1}|}. \end{aligned} \tag{9}$$

The above radius corresponds to the steady joint angle

$$\begin{aligned} \beta_i^*(R_0) &= \arctan\left(\frac{L_i R_{i-1}\mathrm{c}\bar{\gamma}_{i-1} + R_i^* L_{hi} - R_i^* R_{i-1}\mathrm{s}\bar{\gamma}_{i-1}}{R_i^* R_{i-1}\mathrm{c}\bar{\gamma}_{i-1} - L_i L_{hi} + L_i R_{i-1}\mathrm{s}\bar{\gamma}_{i-1}}\right) \\ &= \arctan\left(\frac{L_i R_0\mathrm{c}\bar{\gamma}_{i-1} + R_i^* L_{hi} - R_i^* R_0\mathrm{s}\bar{\gamma}_{i-1}}{R_i^* R_0\mathrm{c}\bar{\gamma}_{i-1} - L_i L_{hi} + L_i R_0\mathrm{s}\bar{\gamma}_{i-1}}\right) \end{aligned} \tag{10}$$

highlighted in red in Fig. 2. Formula (10) has been derived using relation (2) for the steady circular motion conditions taking $\omega_i = \omega_{i-1} = \Omega = \text{const}$, $\gamma_{i-1} = \bar{\gamma}_{i-1} = \text{const}$, and $\gamma_i \equiv 0$.

To find the steady steering angle $\bar{\gamma}_i$ for $R_i = R_{i-1} = R_0(\bar{\beta}_0)$, one have to derive first the steady joint angle $\bar{\beta}_i$ highlighted in blue in Fig. 2. Upon the figure one can write

$$\bar{\beta}_i = \beta_i^*(R_0(\bar{\beta}_0)) - \mathrm{sign}(R_0)(\chi_i - \psi_i), \tag{11}$$

where

$$\chi_i = 2\arctan\left(\frac{2\rho_i(R_0)}{L_i - |R_0| + \sqrt{R_i^{*2} + L_i^2}}\right),$$
$$\psi_i = \arctan(|R_i^*|/L_i),$$

with $\rho_i = \left((\nu_i - L_i)(\nu_i - |R_0|)(\nu_i - \sqrt{R_i^{*2} + L_i^2})/\nu_i\right)^{1/2}$ and $\nu_i = (L_i + |R_0| + \sqrt{R_i^{*2} + L_i^2})/2$. Now, recalling again transformation (2), which for the circular motion conditions with $R_i = R_{i-1} = R_0(\bar{\beta}_0)$ can be rewritten by taking $\omega_i = \omega_{i-1} = \Omega = \text{const}$, $v_i = v_{i-1} = V = \text{const}$, $\beta_i = \bar{\beta}_i$, and $\gamma_{i-1} = \bar{\gamma}_{i-1}$, one can easily find a formula for $\bar{\gamma}_i$ in the form

$$\bar{\gamma}_i(\bar{\beta}_0) = \arctan\left(\frac{L_i + L_{hi}\mathrm{c}\bar{\beta}_i - R_0(\bar{\beta}_0)\mathrm{s}(\bar{\beta}_i + \bar{\gamma}_{i-1})}{-L_{hi}\mathrm{s}\bar{\beta}_i - R_0(\bar{\beta}_0)\mathrm{c}(\bar{\beta}_i + \bar{\gamma}_{i-1})}\right) \tag{12}$$

with $\bar{\beta}_i$ resulting from (11). (12) is a recursive formula which for $i = 1$ should be evaluated by taking $\bar{\gamma}_{i-1} = \bar{\gamma}_0 \equiv 0$ (by assumption of the fixed rear tractor's wheel). Note that (12) shall be read as $\bar{\gamma}_i(R_0(\bar{\beta}_0))$, that is, (12) is constant for any steady value $\bar{\beta}_0$ of angle β_0.

Remark 1. It is worth stressing that the formulas provided in (9) and (10) generalize the classical steady-motion formulas known from the literature for the case of non-steerable tractor wheels – see, e.g., Michałek (2013).

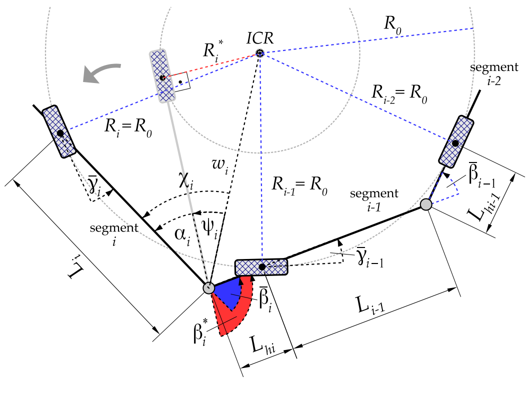

Fig. 2. Geometrical relations in steady-motion conditions along a circle of positive radius $R_0(\bar{\beta}_0) = L_0/\tan\bar{\beta}_0$.

4. STEERING CONTROL SYSTEM

4.1 Control law design

Let us assume for simplicity that $S = N$ (all trailers are equipped with steerable wheels). First, we are going to design a reference-generating function. Let us introduce a steering-to-joint angular ratio (valid for $|\beta_0| > 0$)

$$d_i(\beta_0(t)) \triangleq \frac{\bar{\gamma}_i(\beta_0(t))}{\bar{\beta}_i(\beta_0(t))}, \quad i = 1, \ldots, N \tag{13}$$

which computes a ratio of a trailer steering angle and a joint angle for the ith segment upon the formulas (12) and (11) evaluated, however, at the current steering angle $\beta_0(t)$. Definition (13) reflects the fact that a steady value $\bar{\beta}_0$ along a roundabout is generally unknown in advance, thus one proposes here to compute the ratio with a current steering angle and treat it as it would be a steady value (along a roundabout, we expect that a driver quickly set β_0 to a steady value $\bar{\beta}_0$ leading to $d_i(\bar{\beta}_0) = \text{const}$).

Next, to address transient effects of the trailers, we propose to define the ith reference steering angle

$$\gamma_{id}(t) \triangleq d_i(\beta_0(t)) \cdot \beta_i(t - \tau_i(t)), \quad i = 1, \ldots, N, \qquad (14)$$

using the dynamically delayed ith joint angle, where $\tau_i \geq 0$ is a delay time. We propose to use the following *delay scheduler*

$$\tau_i(t) \triangleq c_i \frac{(L_{hi} + L_i)}{v_i(t)}, \quad c_i > 0, \quad i = 1, \ldots, N, \qquad (15)$$

where c_i is a design coefficient, whereas

$$v_i(t) \stackrel{(2)}{=} [0\ 1] \prod_{j=i}^{1} \boldsymbol{J}_j(\beta_j(t), \gamma_j(t), \gamma_{j-1}(t)) \boldsymbol{u}_0(t)$$

is a longitudinal velocity of the ith trailer, being a scheduling variable in the formula (15). Rationale for introducing the delay time $\tau_i(t)$ in (14) and the scheduling rule (15) will be explained in Section 4.2.

Recalling the integrating nature of the steering dynamics $\dot{\gamma}_i = \zeta_i$ (cf. (6)), let us propose a steering control law

$$\zeta_i(t) \triangleq k_{pi} [\gamma_{id}(t) - \gamma_i(t)] + \dot{\gamma}_{id}(t), \quad k_{pi} > 0, \qquad (16)$$

where k_{pi} is a design parameter, whereas

$$\dot{\gamma}_{id} \stackrel{(14)}{=} \underbrace{\frac{\mathrm{d}d_i(\beta_0)}{\mathrm{d}\beta_0} \zeta_0 \beta_i(t - \tau_i)}_{\zeta_{i\mathrm{FF1}}} + \underbrace{d_i(\beta_0) \frac{\mathrm{d}\beta_i(t - \tau_i)}{\mathrm{d}t}}_{\zeta_{i\mathrm{FF2}}} \qquad (17)$$

is a feedforward term (we used $\dot{\beta}_0 = \zeta_0$, see (6)). For all $i \in \{1, \ldots, N\}$, we can rewrite (16) with (14) in the form

$$\boldsymbol{\zeta}(t) \triangleq \boldsymbol{K}_p [\boldsymbol{D}(\beta_0(t)) \boldsymbol{\beta}_{\mathrm{del}}(t) - \boldsymbol{\gamma}(t)] + \dot{\boldsymbol{\gamma}}_d(t), \qquad (18)$$

where $\boldsymbol{K}_p = \mathrm{diag}\{k_{pi}\}$, $\boldsymbol{D}(\beta_0(t)) = \mathrm{diag}\{d_i(\beta_0(t))\}$ for $i = 1, \ldots, N$, and $\boldsymbol{\beta}_{\mathrm{del}}(t) \triangleq [\beta_1(t - \tau_1(t)) \ldots \beta_N(t - \tau_N(t))]^\top$. A block scheme of the proposed control system has been presented in Fig. 3. It is evident that application of (16)

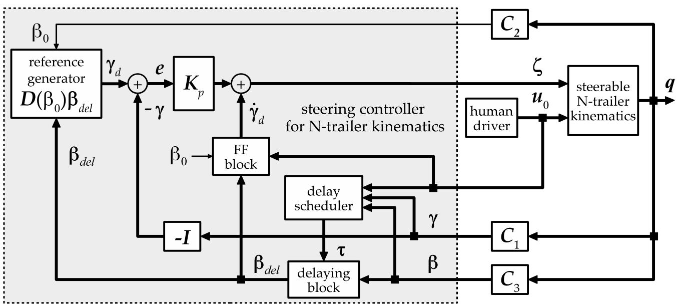

Fig. 3. Block scheme of the proposed control system.

into steering dynamics $\dot{\gamma}_i = \zeta_i$ leads to the steering-error equation $\dot{e}_i(t) + k_{pi} e_i(t) = 0$, where $e_i \triangleq \gamma_{id} - \gamma_i$, implying $e_i(t) \to 0$ as $t \to \infty$.

Remark 2. When negotiating a roundabout, $\beta_0(t)$ is quickly set to a steady value $\bar{\beta}_0$ by a driver. In this case $\zeta_0 = 0$, and

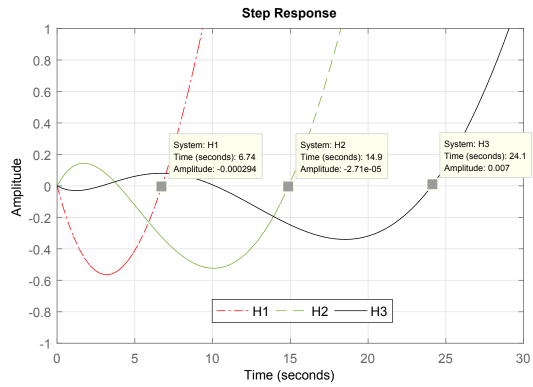

Fig. 4. Initial undershoots and oscillations in step responses of nonminimum-phase transfer functions H_i.

implementation of (17) can be reduced to term $\zeta_{i\mathrm{FF2}}$. Such a simplification was used in simulations (see Section 5) leading to the transients of $|e_i(t)|$ on a level of 10^{-5} rad.

Remark 3. By combining (14) with (13) one may write $\gamma_{di}(t) = \bar{\gamma}_i(\beta_0(t)) \frac{\beta_i(t-\tau_i(t))}{\bar{\beta}_i(\beta_0(t))}$. This form can be interpreted as a *predicted steady* steering angle $\bar{\gamma}_i$ multiplied by a delayed *shaping function* $\frac{\beta_i(t-\tau_i(t))}{\bar{\beta}_i(\beta_0(t))}$. If $\beta_i(t) \to \bar{\beta}_i$, as $\beta_0(t) \to \bar{\beta}_0$, then $\gamma_{di}(t)$ tends to $\bar{\gamma}_i(\bar{\beta}_0)$ resulting from (12). The shaping function is bounded if $|\beta_0(t)| > 0$. To avoid singularities, it suffices to select a threshold $0 < \beta_\epsilon \ll |\bar{\beta}_0|$, and for $|\beta_0(t)| < \beta_\epsilon$ evaluate (13) taking $d_i(\beta_\epsilon \mathrm{sgn}(\beta_0(t)))$, where $\mathrm{sgn}(z) = 1$ if $z \geq 0$ and $\mathrm{sgn}(z) = -1$ if $z < 0$.

4.2 Rationale for (15) and tuning of coefficients c_i

According to Michałek (2013), transfer functions (obtained for model (6) when linearised in the straight-line motion conditions) between orientation angles θ_i of the trailers and the angular velocity ω_0 of the tractor take the forms

$$H_i(s) \triangleq \frac{\Theta_i(s)}{\Omega_0(s)} = \frac{1}{s} \prod_{j=1}^{i} F_j(s), \quad F_j(s) = \frac{(1 - \frac{L_{hj}}{\bar{v}_0} s)}{(1 + \frac{L_j}{\bar{v}_0} s)}, \qquad (19)$$

where s is a complex variable, and $\bar{v}_0 = \text{const} > 0$ is a vehicle longitudinal velocity. $F_i(s)$ represents nonminimum-phase dynamics if $L_{hi} > 0$. In this case, one shall expect either undershoot or/and initial oscillations in a time response of $\theta_i(t)$ when entering (and exiting) a roundabout. Application of (14) with $\tau_i \equiv 0$ could amplify the mentioned nonminimum-phase effects and worsen a resultant transient control performance. Figure 4 illustrates exemplary step responses of dynamics $H_1(s)$, $H_2(s)$, and $H_3(s)$. An average residence time for $F_i(s)$ can be approximated by $(L_{hi} + L_i)/\bar{v}_0$. The form of (15) results directly from this kind of approximation, including the additional tuning coefficient c_i and using the scheduling variable $v_i(t)$ (instead of the constant velocity $\bar{v}_0$) to dynamically scale $\tau_i(t)$ during the transient motion. One can also treat (15) as an approximated time interval needed for point (x_i, y_i), moving with velocity $v_i(t)$, to reach a position of a preceding point (x_{i-1}, y_{i-1}) along a path drawn by the latter.

We propose to select the coefficient c_i in (15) by a simple heuristics, namely, by satisfying the following equation

$$c_i \frac{(L_{hi} + L_i)}{v_0} = T_i / \mu_i, \quad \mu_i \geq 1,$$

where v_0 is applied by a driver (note: $v_i(t) \to v_0$ for a steady circular motion with the zero SOT), and $T_i \geq 0$ is a time instant of the last zero-crossing read out from a step response of $H_i(s)$ (cf. Fig. 4), whereas μ_i is a scaling factor. For the first two trailers a good choice is $\mu_1 = \mu_2 = 1$. Selection of μ_i for $i > 2$ is usually not difficult, but it has to be chosen by trials and errors (no formal rule for this selection is known thus far). Usage of the coefficient c_i and the factor μ_i helps one to scale $\tau_i(t)$ for a beneficial vehicle behaviour with respect to MEnS and MExS.

5. SIMULATION RESULTS

The proposed steering controller has been validated with a 21.5-meter long multi-steering 3-trailer vehicle equipped solely with off-axle hitching (the so-called non-Standard 3-Trailer, or nS3T kinematics). Thus, in this case $S = N$ implying $\boldsymbol{\gamma} = [\gamma_1\ \gamma_2\ \gamma_3]^\top$ and $\boldsymbol{\zeta} = [\zeta_1\ \zeta_2\ \zeta_3]^\top$. This structure belongs to the most demanding kinematics due to the appearance of nonminimum-phase transient effects resulting from the presence of the off-axle interconnections, see Michałek (2013). The kinematic parameters of the vehicle and the design factors have been selected as follows: $L_0 = 5.0\,\text{m}$, $L_1 = 4.0\,\text{m}$, $L_2 = 3.0\,\text{m}$, $L_3 = 5.0\,\text{m}$, $L_{hi} = 1.5\,\text{m}$ and $k_{pi} = 20.0$ for $i = 1, 2, 3$. A 450° roundabout maneuver with the constant tractor speed $v_0 = \bar{v}_0 = 0.4\,\text{m/s}$ has been performed, corresponding to the steady steering angle $\bar{\beta}_0 = 0.5\,\text{rad}$. The delay coefficients have been computed for $\mu_{1,2} = 1$ and $\mu_3 = 3$ as: $c_1 = 0.48$, $c_2 = 1.33$, $c_3 = 0.49$. The control performance obtained with control law (18), labelled in Table 1 as *delayed steering*, has been compared with a result of steering with $\tau_i(t) \equiv 0$ for all i (labelled as *steering*), and with the case of towed trailers with non-steerable wheels (labelled as *no steering*). Obtained values of measures SSPW, SOT, MEnS, and EMxS for particular cases are presented in Table 1. Figure 5 illustrates the vehicle behaviour for all three cases, while time-plots in Fig. 6 show the evolution of selected signals in a closed-loop system with proposed control law (18). The widths of swept steady paths in a case of *delayed steering* control and in the case of *no steering* have been compared in Fig. 7.

Table 1. Values of performance measures obtained in simulations for the nS3T kinematics.

	delayed steering	steering	no steering
SSPW [m]	4.95	4.95	6.93
SOT [m]	0.00	0.00	2.79
MEnS [m]	0.23	0.72	0.04
MExS [m]	0.00	0.00	0.00

Upon the results, one may conclude that delaying the reference signals is crucial to improve a transient response of the vehicle – it allows avoiding large amplification of the nonminimum-phase effects on a roundabout entrance (reflected here by the MEnS measure, which is more than three time smaller for the case of *delayed steering* relative to the *steering* case), and allows decreasing substantial cut-in effects on exiting a roundabout (extensive cut-in is visible in Fig. 5 B). Although the value of MEnS it larger for the *delayed steering* than for the *no steering* case, substantial improvement of SSPW by about 2 m obtained thanks to the *delayed steering* controller is evident in this comparison.

6. FINAL REMARKS

The proposed steering controller is generic – it can be applied to any N-trailer composed of a car-like tractor and arbitrary number of segments interconnected with arbitrary type of hitching. The control law seems to be relatively simple in practical implementation because it uses a feedback from internal (easily measurable) angular variables of a vehicle. Key properties of the controller come from the closed-form recursive formulas derived for an angular configuration in the circular motion conditions, and from appropriate delaying of reference steering signals with dynamically scheduled delay times. A transient control performance depends on how well the coefficients c_i are tuned. Selection of c_i to minimize undesirable swing effects for any structure of the N-trailer still remains an open issue. A potential weakness of the proposed controller, coming from a slow convergence rate to steady motion conditions in the case of long trailers, can be probably improved by modifying the form of shaping function (13). The latter two problems, together with a robustness analysis of the closed-loop system to model and feedback perturbations, are interesting topics for further investigations.

ACKNOWLEDGEMENTS

This work was supported in part by the National Science Centre, Poland, under the grant 2016/21/B/ST7/02259, and in part by the research subvention 0211/SBAD/0911.

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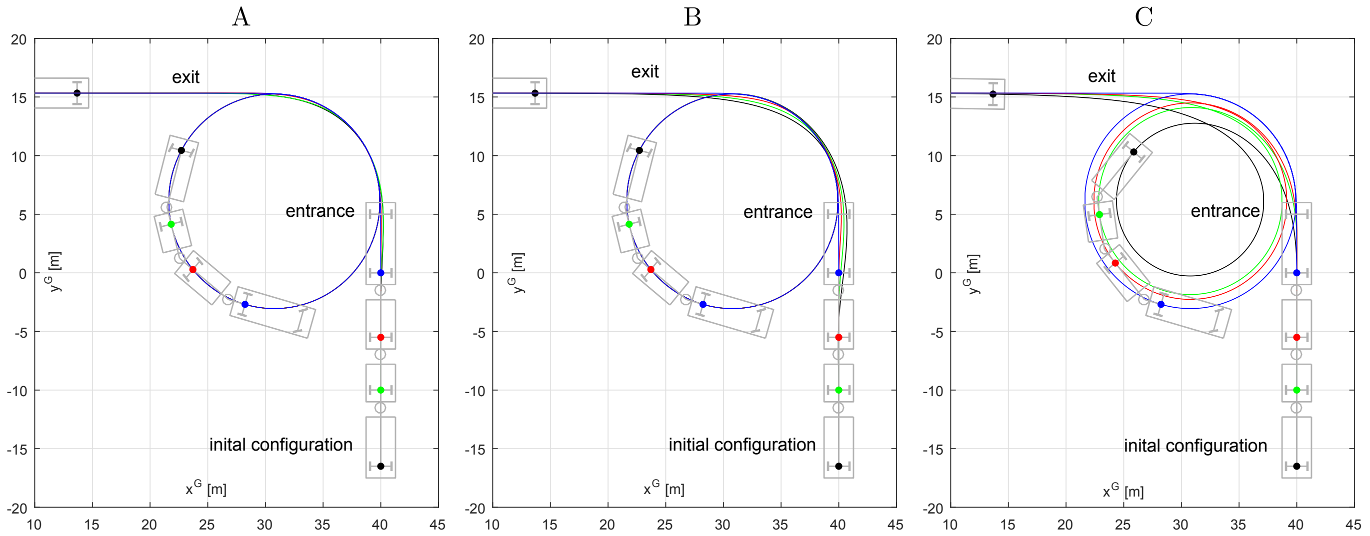

Fig. 5. Roundabout maneuvering with the nS3T vehicle: with delayed steering (A), with steering and no delay (B), and with no steering (C); the path drawn by the characteristic point of the tractor has been highlighted in blue.

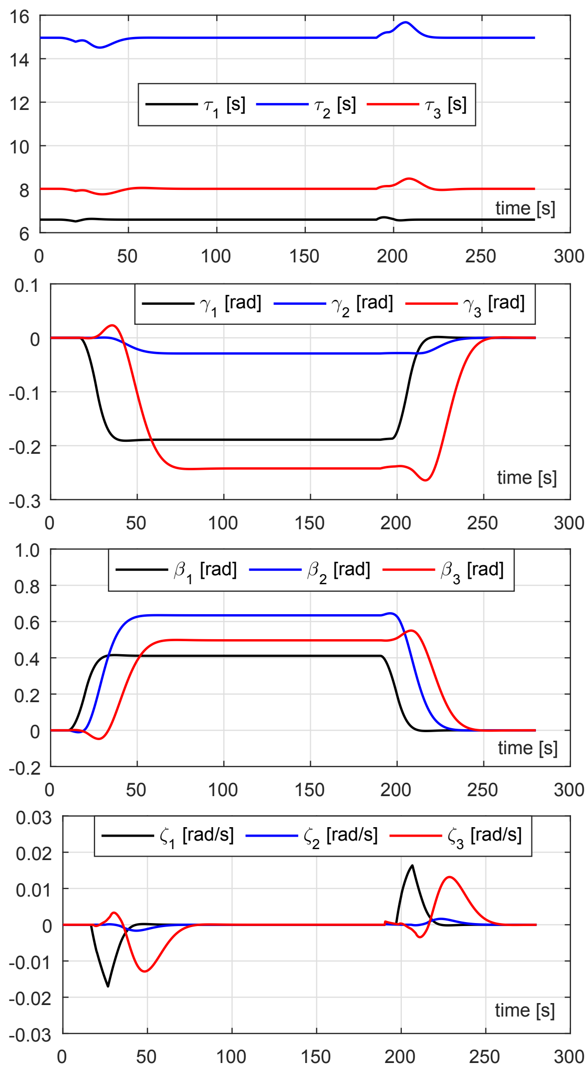

Fig. 6. Time plots of selected signals in the case of delayed steering control obtained for the nS3T vehicle.

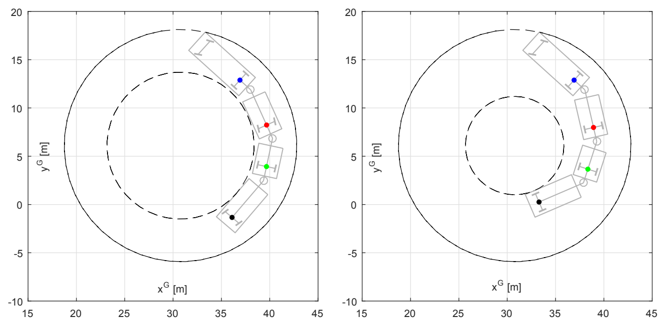

Fig. 7. The paths swept by the nS3T vehicle in the steady motion conditions along a roundabout for the *delayed steering* case (left) and for the *no steering* case (right).

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