The concept of passive control-assistance for docking maneuvers with N-trailer vehicles

Maciej Marcin Michałek, *Member, IEEE*, Marcin Kiełczewski

***Abstract*—We present a concept of passive control-assistance system which can help a human driver in precise maneuvers with a tractor-trailers vehicle in the task of docking with the last trailer. The novel approach is developed for *truly* N-trailer vehicles comprising a car-like tractor and *arbitrary* number of on-axle or off-axle hitched trailers. Passivity of the proposed assistance system results from the fact that it does not interact directly with a vehicle, but acts solely as an advisor suggesting control action to a human operator through a passive human-machine interface (HMI). The key role in the concept plays the cascaded Vector-Field-Orientation (VFO) feedback control law responsible for computation of the efficient control strategy for a driver based on a feedback from a current vehicle configuration. The passive assistance system has been functionally compared with an alternative active control-assistance proposed in the literature. The paper reports the results of experimental tests conducted with a laboratory-scale vehicle, which illustrate efficacy of the cooperation between a driver and a control assistant in the task of backward docking with three trailers.**

***Index Terms*—N-trailer, feedback control, docking maneuvers, driver-assistance-system, control-assistance, HIL, HMI**

I. Introduction

Precise maneuvering with N-trailer vehicles (N-trailers) belongs to non-intuitive, difficult, and burdening tasks even for experienced human drivers [5], [19], [38], [41]. Ones of the hardest motion tasks executed with N-trailers are backward parking maneuvers, where the last trailer must be precisely positioned at the desired location (the task of docking with a trailer) [4], [25], [38], [42]. Essential difficulties in maneuvering come from specific kinematic properties characteristic for the N-trailers [1], [10], [20], [24], [27], [35]; difficulties substantially increase with a number of trailers attached to a tractor. In this paper the problem of precise docking is addressed by application of a control-assistance system [15], which could effectively cooperate with a human driver helping him/her apply appropriate control actions to smoothly execute required maneuvers with arbitrary number of trailers.

So far, parking-assistance systems have been widely proposed for the single-body car-like vehicles (see e.g. [7], [18], [40]), and have become now commercially available [36]. Recently, the parking assistance system has been also proposed for the marine vehicles, see e.g. [16]. However, their counterparts for the multi-body tractor-trailer vehicles are still in a phase of laboratory tests, and are usually restricted to vehicles with a strictly limited number of trailers. An example of the control-assistance system devised for a commercial truck towing a full-trailer has been presented in [37], [38]; other examples of various kinds of specialized assistance systems for tractor-trailer vehicles were addressed e.g. in [3], [8], [32].

Authors are with Chair of Control and Systems Engineering, Poznań University of Technology (PUT), Piotrowo 3a, 60-965 Poznań, Poland, e-mail: maciej.michalek@put.poznan.pl

In contrast to the automatic vehicular guidance systems, which completely replace a driver making the vehicle a robotic system [6], [21], [23], [25], [26], [33], [39], we will consider the human-in-the-loop (HIL) control concept, where a driver and an automatic assistant cooperate together to achieve the control objectives. In the HIL system a human driver applies the control action by using a conventional mechanical interface and still takes the entire responsibility for the control process treating the control assistant only as an advisor. Such an approach does not rise difficult and still unresolved legal issues concerning the case where robots and humans share a common task space. Control-assistance solutions proposed in the literature for the N-trailers are mostly focused on the *active* assistance concept [22], [30], [31], [34], where a tractor must be equipped with the expensive steer-by-wire implement and the cruise control system. These requirements restrict practical application of the active methods in commercial vehicles equipped only with a conventional manual steering wheel and a speed/acceleration pedal. An alternative and cheaper solution is the *passive* assistance which is free of the mentioned limitations and can be applied into the N-trailers with conventional tractors. Description of working principles and functional comparison of active and passive control-assistance systems are provided in Section II-B.

In this paper we present a passive control-assistance system for the task of docking developed for *truly* N-trailers comprising a car-like tractor and *arbitrary* number of single-axle trailers interconnected by on-axle or off-axle rotary joints. A core of the proposed system is the cascaded Vector-Field-Orientation (VFO) feedback control strategy introduced and formally analyzed in [9], [12], which plays a role of the control assistant. Taking into account naturally limited perception of a human driver we will show that control actions computed by the passive assistant can be efficiently suggested to a driver by a minimalistic graphical Human-Machine Interface (HMI). The latter property makes the proposed assistance system relatively easy to use, enabling successful completion of precise docking maneuvers even for unexperienced drivers. The paper is an extension of our prior conference article [11].

II. The concept of control-assistance and control objective formulation

A. Model of the N-trailer

We will restrict modeling of the N-trailer solely to the kinematic level which corresponds to the so-called *low speed* *steering* motion [17]. This simplification is motivated by two practical reasons: 1) precise docking maneuvers are usually executed with small velocities to keep safety and feasibility of the task, thus the effects caused by vehicle dynamics are secondary and often negligible in this case, 2) most difficulties with docking maneuvers have their origins just in specific properties of the N-trailer kinematics which impose sophisticated constraints on the vehicle motion.

Let us consider the N-trailer vehicle comprising a front-axle driven car-like tractor (segment number zero) and a number of N trailers interconnected in a chain by the passive rotary joints (cf. Fig. 1). We assume that wheel axles of all the trailers are non-steerable (fixed) and passive (non-actuated). The tractor is the only active vehicle segment with the control input

$$\boldsymbol{u}_{\mathrm{F0}} \triangleq [\omega_{\mathrm{F0}} \;\, v_{\mathrm{F0}}]^\top \in \mathbb{R}^2, \tag{1}$$

where $\omega_{\mathrm{F0}} = \dot{\beta}_0$ is a steering velocity of a front tractor wheel, and v_{F0} is a longitudinal velocity of a middle point of the front tractor wheel (Fig. 1). Further, we assume that in the case of a conventional vehicle controlled by a human driver steering velocity ω_{F0} can be forced by turning a steering wheel, while velocity v_{F0} can be forced by a pedal. Let us also distinguish the so-called *tractor-body velocities*, ω_0 and v_0, which are related with control input (1) by the well-known relations

$$v_0 = v_{\mathrm{F0}} \cos\beta_0, \quad \omega_0 = \frac{1}{L_0} v_{\mathrm{F0}} \sin\beta_0, \quad \dot{\beta}_0 = \omega_{\mathrm{F0}}, \tag{2}$$

where $L_0 > 0$ denotes a tractor length, and β_0 is a steering angle of the front tractor wheel (see Fig. 1). Equations (2) will be used in subsequent considerations.

For the purpose of a motion task definition we distinguish the last vehicle trailer calling it the *guidance segment* with the guidance point $P_N = (x_N, y_N)$ located at the midpoint of the wheels axle. The vehicle is characterized by two types of kinematic parameters (see Fig. 1): trailer lengths $L_i > 0$ and hitching offsets $L_{hi} \in \mathbb{R}$, $i = 1, \ldots, N$, where for $L_{hi} \neq 0$ one says about the off-axle hitching and for $L_{hi} = 0$ about the on-axle one [1], [9]. The offset L_{hi} is treated as positive when the ith hitching point is located *behind* a preceding wheel axle, and negative in the opposite case. From now on we assume that

A1. $\forall\, l, j: \;\; L_{hl} \neq 0 \;\wedge L_{hj} \neq 0 \;\Rightarrow\; L_{hl}L_{hj} > 0.$

Assumption A1 means that all the non-zero hitching offsets in a vehicle must have a common sign. Although A1 may seem limiting, most practical constructions of N-trailers satisfy this assumption. In the paper we will consider in particular two practically meaningful kinematic structures of N-trailer vehicles called in the literature as non-Standard N-Trailers (nSNT) and Generalized-N-Trailers (GNT). The former are equipped solely with off-axle joints, while the latter possess mixed on-axle and off-axle hitches (for classification of N-trailer structures the reader is referred to [1], [10], [24]).

Configuration of the N-trailer body can be uniquely determined by $N+3$ variables (cf. [1], [10], [24]) which comprise a posture (position coordinates x_i, y_i and orientation θ_i) of an arbitrarily selected vehicle segment (tractor or one of the trailers) and all the joint angles $\beta_1, \ldots, \beta_N$ which determine a *shape* of

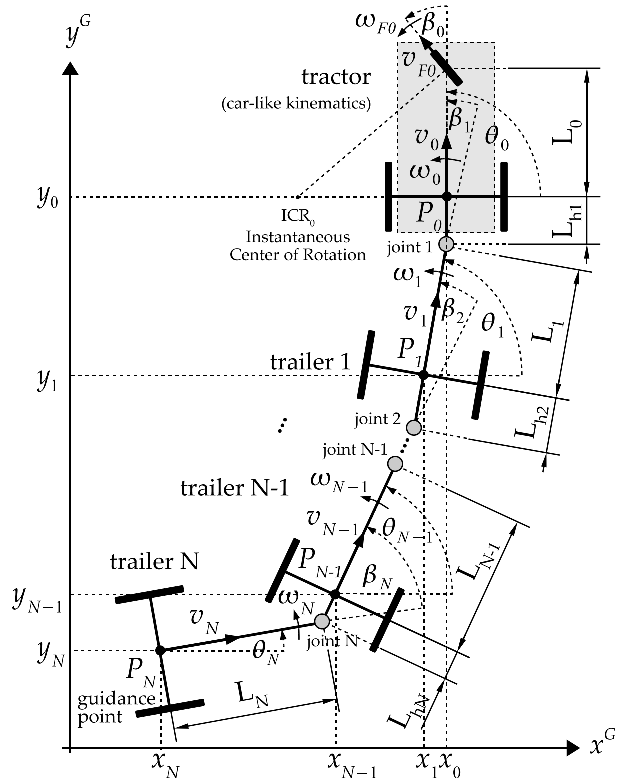

Fig. 1. N-trailer vehicle in a global frame – kinematic structure with definition of configuration variables and control inputs.

a vehicle chain[1]. Motivated by the control objective considered in the sequel – see the formulation in Section II-C – we select a posture of the guidance segment, $\boldsymbol{q}_N = [\theta_N \; x_N \; y_N]^\top \in \mathbb{R}^3$, as a part of the N-trailer configuration vector

$$\boldsymbol{q} \triangleq \begin{bmatrix} \boldsymbol{\beta} \\ \boldsymbol{q}_N \end{bmatrix} = [\beta_1 \;\ldots\; \beta_N \;\; \theta_N \; x_N \; y_N]^\top \in \mathbb{T}^N \times \mathbb{R}^3 \tag{3}$$

where $\boldsymbol{\beta} \in \mathbb{T}^N$ denotes a sub-vector of joint-angles (geometrical interpretation of configuration variables results from the kinematic scheme presented in Fig. 1). Configuration vector (3) lets one uniquely determine position and orientation of any vehicle segment in a global frame by applying basic geometrical relationships (see [24]) based upon the scheme presented in Fig. 1.

B. General concepts of active and passive control-assistance

In Figs. 2 and 3 have been presented functional block schemes of the two alternative – ACTIVE and PASSIVE – control-assistance systems proposed so far in the literature for the N-trailers. Both schemes utilize feedback from the current vehicle configuration $\boldsymbol{q}$ which is assumed to be measurable (or can be estimated by using proprioceptive and exteroceptive sensory systems). Let us briefly explain and compare the two concepts.

[1]Alternatively to angles $\beta_1, \ldots, \beta_N$ one could select here orientation angles θ_i of all the vehicle segments, however measuring the joint angles is much easier in practice.

In the ACTIVE control assistance, applied e.g. in [22], [29], [34], a human operator is responsible for on-line determination of a motion strategy for the guidance segment of a vehicle in order to complete the stated motion task (represented on the scheme by the control objective). By the available mechanical interface a human operator commands suggested velocities for the guidance segment to the active assistant block. Upon the current configuration $\boldsymbol{q}$ (provided through the internal feedback loop depicted in Fig. 2) the active assistant on-line transforms commanded velocities into desired velocities for the tractor, and applies them to the tractor control inputs. Since the assistant block directly affects the vehicle input and forces a vehicle motion, the assistance scheme is called active. A purpose of the HMI subsystem is to provide feedback information on configuration $\boldsymbol{q}$ in a form suitable for a human operator (i.e. by the visual, audio or haptic forms [2], [15], [18]). This information allows the operator to modify the motion strategy for a guidance segment and keep safety of the maneuvers. Application of the ACTIVE control-assistance system in a conventional tractor-trailer vehicle require installation of the steer-by-wire and cruise control systems to enable automatic execution of desired tractor velocities.

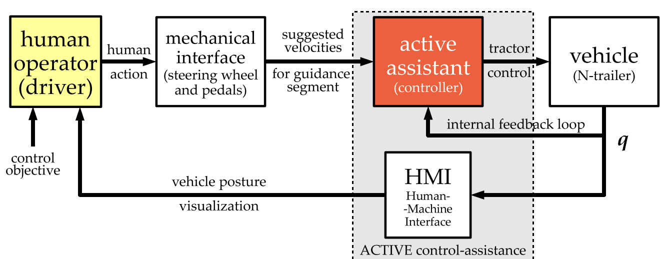

Fig. 2. Functional block scheme of the ACTIVE control-assistance system.

In contrast to the above scheme, the PASSIVE control-assistance system, proposed by the authors for the first time in [11], utilizes a conventional way of affecting the tractor motion by using the classical mechanical interface in the form of a steering wheel and a speed pedal. In this case, the passive assistant block in Fig. 3 consists of an appropriately selected feedback control law devised for N-trailers. Upon the control objective and current configuration $\boldsymbol{q}$ the passive assistant computes instantaneous suggestions for the tractor velocities, which should be forced by the human operator in order to meet the stated motion task (represented on the scheme by the control objective). Computed suggestions are then converted into the form of expected instantaneous manual/pedal actions and are provided to a human operator by the HMI module. The auxiliary feedback loop denoted in Fig. 3 helps the human operator monitor and, if necessary, correct his/her actions by comparing them with suggestions provided by the HMI. Worth stressing that in this case the assistant block does not interact directly with a tractor [36], hence all the responsibility of motion execution is left to a human operator who may either respect or discard the assistant suggestions at any time instant. This property explains why the assistance system is called passive. Effectiveness of the PASSIVE assistance scheme essentially depends on the two components: the control law applied in the assistant block, and the form in which control suggestions are provided to a human operator.

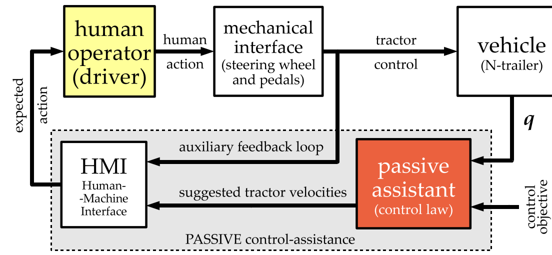

Fig. 3. Functional block scheme of the PASSIVE control-assistance system.

Table I summarizes essential properties of the ACTIVE and PASSIVE control-assistance schemes available for the N-trailers. One can find substantial differences between the two approaches, where the main distinction results from the swap of roles played by the assistant and a human operator in particular control schemes.

C. Control objective formulation

Let us focus on the PASSIVE control-assistance system for the task of docking, which is the main topic of the paper. We need to formally state the control objective which should be achieved by the control strategy employed in the assistant block. For this purpose we define the output of vehicle kinematics

$$\boldsymbol{y} \triangleq \boldsymbol{q}_N = \boldsymbol{C}\boldsymbol{q}, \quad \boldsymbol{C} = [\boldsymbol{0}_{3\times N}\ \operatorname{diag}\{1,1,1\}_{3\times 3}] \tag{4}$$

being a posture of the guidance segment. The task of docking with the last trailer can be formulated by introducing the fixed reference point

$$\boldsymbol{y}_d \triangleq \boldsymbol{q}_{Nd} = [\theta_{Nd}\ x_{Nd}\ y_{Nd}]^\top \in \mathbb{R}^3 \tag{5}$$

which determines desired orientation θ_{Nd} and position $\bar{\boldsymbol{q}}_{Nd} = [x_{Nd}\ y_{Nd}]^\top$ of the dock, where the guidance segment should be positioned. By definition of the output error

$$\boldsymbol{e} = \begin{bmatrix} e_\theta \\ e_x \\ e_y \end{bmatrix} \triangleq \boldsymbol{y}_d \ominus \boldsymbol{y} \triangleq \begin{bmatrix} \mathcal{F}(\theta_{Nd}-\theta_N) \\ x_{Nd}-x_N \\ y_{Nd}-y_N \end{bmatrix} \in (-\pi,\pi]\times\mathbb{R}^2 \tag{6}$$

with $\mathcal{F} : \mathbb{R} \mapsto (-\pi,\pi]$, we introduce the weighted posture error

$$\boldsymbol{e}_w \triangleq \boldsymbol{W}\boldsymbol{e}, \qquad \boldsymbol{W} = \operatorname{diag}\{w,1,1\}, \tag{7}$$

with weight $w \in [0,1]$ which allows selecting a proportion between particular error components of different units. The control objective is to provide for every $t \geq 0$ suggested tractor-body velocities

$$\boldsymbol{u}_{0s}(t) = \begin{bmatrix} \omega_{0s}(t) \\ v_{0s}(t) \end{bmatrix} \in \mathbb{R}^2, \tag{8}$$

which guarantee, when forced (i.e. for $\omega_0(t) = \omega_{0s}(t)$ and $v_0(t) = v_{0s}(t)$), that

O1. $\forall\, t \geq 0\ \ \|\boldsymbol{e}_w(t)\|, \|\boldsymbol{\beta}(t)\| < \infty$,
O2. $\exists\, T(\delta,\cdot) \in [0,\infty) :\ \forall\, t \geq T\ \|\boldsymbol{e}_w(t)\| \leq \delta$,

TABLE I
ESSENTIAL PROPERTIES COMPARISON OF THE ACTIVE AND PASSIVE CONTROL-ASSISTANCE SYSTEMS

PROPERTY	ACTIVE control-assistance	PASSIVE control-assistance
1. Assistant role	fulfills suggestions of the human operator (inner-loop controller)	suggests velocities for the tractor (outer-loop control strategy)
2. Human operator role	suggests velocities for the guidance segment (outer-loop control strategy)	fulfills suggestions of the assistant (inner-loop controller)
3. Component directly affecting the tractor motion	assistant block	human operator
4. Installation of steer-by-wire and cruise control systems	required	NOT required
5. Measurement of configuration $\boldsymbol{q}$	required	required
6. HMI implementation	required	required

where $T(\delta, \cdot)$ is called the docking time-horizon, and $\delta \geq 0$ is a prescribed docking accuracy.

O1 requires boundedness of all the configuration variables during maneuvering with the N-trailer. O2 reflects the expectation according to which the weighted output error converges to the prescribed vicinity of zero in time horizon $T(\delta, \cdot)$. As a consequence, objectives O1 and O2 restrict a class of control laws which can be used in the assistant block. In Section III-A we propose the cascaded VFO control laws introduced in [9] and [12], which turned out to be especially effective in achieving the objectives O1 and O2. In Section III-B we show how the suggested tractor-body velocities (8) can be converted into a single steering suggestion for a human driver of a car-like tractor with input (1).

III. PASSIVE CONTROL-ASSISTANCE FOR N-TRAILERS

A. Cascaded VFO control law as the control assistant

We propose to design the control-assistance block by utilization of the cascaded VFO control laws introduced for the set-point control task in papers [9] and [12]. Let us recall origins of the control law and explain the overall cascaded VFO control strategy to make further considerations clear enough.

Looking at the multi-body structure presented in Fig. 1, and assuming the rolling-without-skid motion conditions[2] for all the vehicle wheels, one can treat the N-trailer as a chain of unicycle-like bodies interconnected by the passive joints. Denoting by $\boldsymbol{u}_i = [\omega_i \; v_i]^\top \in \mathbb{R}^2$ a velocity vector of the ith vehicle body in the chain one may describe kinematics of the ith vehicle segment by equation

$$\begin{bmatrix} \dot{\theta}_i \\ \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \cos\theta_i \\ 0 & \sin\theta_i \end{bmatrix} \begin{bmatrix} \omega_i \\ v_i \end{bmatrix} = \boldsymbol{G}(\theta_i)\boldsymbol{u}_i, \quad i = 0, \ldots, N, \quad (9)$$

where $\boldsymbol{q}_i = [\theta_i \; x_i \; y_i]^\top$ denotes the posture, while ω_i and v_i are the angular and longitudinal velocities of the ith segment, respectively (note: for $i = 0$ the tractor-body velocities ω_0, v_0 are related with tractor control inputs through equations (2)). Due to the presence of interconnections between the vehicle bodies any two velocities $\boldsymbol{u}_i$ and $\boldsymbol{u}_{i-1}$ are not independent. According to the basic velocity geometry one can easily find the following transformation

$$\boldsymbol{u}_i = \boldsymbol{J}_i(\beta_i)\boldsymbol{u}_{i-1} = \begin{bmatrix} \frac{-L_{hi}}{L_i}\cos\beta_i & \frac{1}{L_i}\sin\beta_i \\ L_{hi}\sin\beta_i & \cos\beta_i \end{bmatrix} \begin{bmatrix} \omega_{i-1} \\ v_{i-1} \end{bmatrix}, \quad (10)$$

[2]The absence of skid for the wheels of the ith segment is equivalent to satisfaction of the nonholonomic constraint [24]: $\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0$.

which maps velocities between two neighboring segments with matrix $\boldsymbol{J}_i(\beta_i)$ being invertible for any β_i if only $L_{hi} \neq 0$.

Let us consider the guidance segment in the form (9) for $i = N$, with posture $\boldsymbol{q}_N$, and with virtual input $\boldsymbol{u}_N = [\omega_N \; v_N]^\top$. According to the control objective stated in Section II-C we are going to make posture of the guidance segment converge to the reference point (5). Hence, assume there exists some control function $\boldsymbol{\Phi}(\boldsymbol{e}) = [\Phi_\omega(\boldsymbol{e}) \; \Phi_v(\boldsymbol{e})]^\top$, devised for unicycle kinematics, which for $\omega_N := \Phi_\omega(\boldsymbol{e})$ and $v_N := \Phi_v(\boldsymbol{e})$ guarantees that

G1. $\forall t \geq 0 \; \|\boldsymbol{e}(t)\| < \infty$,
G2. $\|\boldsymbol{e}(t)\| \to 0$ with time.

Having such a control function, the key idea is to force $\boldsymbol{\Phi}(\boldsymbol{e})$ on virtual input $\boldsymbol{u}_N$ by appropriate definition of tractor-body velocities. Before providing a definition of the desired tractor-body velocities resulting from the cascaded control concept introduced in [9], [12], let us first determine the form of function $\boldsymbol{\Phi}(\boldsymbol{e})$. To this purpose we will utilize the geometrically motivated VFO control law which turned out to be especially effective in the context of the docking task thanks to the so-called *directing effect* characteristic for motion of a vehicle guided by the VFO controller. The directing effect resembles the parking-to-garage maneuver and it reveals when a vehicle approaches a reference position [9]. Equations of the VFO set-point controller in two versions for the finite-time (F-T) and infinite-time (I-T) convergence can be formulated as follows:

$$\boldsymbol{\Phi}(\boldsymbol{e}) = \begin{bmatrix} \Phi_\omega(\boldsymbol{e}) \\ \Phi_v(\boldsymbol{e}) \end{bmatrix} \triangleq \begin{bmatrix} k_a(\theta_{Na} - \theta_N) + \dot{\theta}_{Na} \\ \rho(e_x, e_y)\cos\alpha \end{bmatrix}, \quad (11)$$

where function $\rho(e_x, e_y) : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}_{\geq 0}$ takes the form

$$\rho(e_x, e_y) \triangleq \begin{cases} (h_x^2 + h_y^2)^{1/2} & \text{for I-T convergence,} \\ (e_x^2 + e_y^2)^{\gamma/2} & \text{for F-T convergence,} \end{cases} \quad (12)$$

while

$$\theta_{Na} \triangleq \begin{cases} \mathrm{Atan2c}\,(\sigma h_y, \sigma h_x) & \text{for} \quad h_x^2 + h_y^2 \neq 0 \\ \theta_{Nd} \mod 2\pi & \text{for} \quad h_x^2 + h_y^2 = 0 \end{cases}, \quad (13)$$

$$\dot{\theta}_{Na} = \begin{cases} \frac{\dot{h}_y h_x - h_y \dot{h}_x}{h_x^2 + h_y^2} & \text{for} \quad h_x^2 + h_y^2 \neq 0 \\ 0 & \text{for} \quad h_x^2 + h_y^2 = 0 \end{cases}, \quad (14)$$

$$h_x \triangleq k_p e_x - \eta\sigma\sqrt{e_x^2 + e_y^2}\cos\theta_{Nd}, \quad (15)$$

$$h_y \triangleq k_p e_y - \eta\sigma\sqrt{e_x^2 + e_y^2}\sin\theta_{Nd}, \quad (16)$$

$$\cos\alpha \triangleq (h_x\cos\theta_N + h_y\sin\theta_N)/\sqrt{h_x^2 + h_y^2}, \quad (17)$$

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with Atan2c $(\cdot,\cdot):\mathbb{R}\times\mathbb{R}\mapsto\mathbb{R}$ being a continuous version of the four-quadrant function Atan2 $(\cdot,\cdot):\mathbb{R}\times\mathbb{R}\mapsto(-\pi,\pi]$ (see e.g. [12] and [13]). In the above definitions four design parameters have been introduced: $k_a, k_p > 0$, $\eta\in(0,k_p)$, $\gamma\in(0,1)$, and decision factor $\sigma\in\{-1,+1\}$. The latter will determine a motion strategy for the guidance segment (backward if $\sigma=-1$ and forward if $\sigma=+1$). Discussion on properties of the VFO control law (11) together with convergence analysis for posture error (6) in a closed-loop system with unicycle kinematics can be found in [14] for the I-T case and in [13] for the F-T case (see also [9] and [12]). For our purposes it is enough to recall that control function (11) satisfies G1 and G2.

Now, let us consider how control function $\boldsymbol{\Phi}(\boldsymbol{e})$ can be forced on the virtual input of the guidance segment. Define the desired velocity for the Nth trailer as

$$\boldsymbol{u}_{Nd}\triangleq\boldsymbol{\Phi}(\boldsymbol{e}). \tag{18}$$

Since (10) is valid for any segment velocities, it holds also for desired velocities $\boldsymbol{u}_{id}=[\omega_{id}\ v_{id}]^\top$ and $\boldsymbol{u}_{i-1d}=[\omega_{i-1d}\ v_{i-1d}]^\top$, i.e.

$$\boldsymbol{u}_{id}=\boldsymbol{J}_i(\beta_i)\boldsymbol{u}_{i-1d},\quad i=1,\dots,N. \tag{19}$$

To determine the inverse transformation to (19) one should separately address two cases: c1) where $L_{hi}\neq0$ (off-axle hitching), and c2) where $L_{hi}=0$ (on-axle hitching). In case c1) one can formulate the inverse velocity transformation as follows [9]

$$\boldsymbol{u}_{i-1d}=\boldsymbol{\Psi}_i^{\text{off}}(\boldsymbol{u}_{id},\beta_i), \tag{20}$$

where mapping

$$\begin{aligned}\boldsymbol{\Psi}_i^{\text{off}}(\boldsymbol{u}_{id},\beta_i)&\triangleq\boldsymbol{J}_i^{-1}(\beta_i)\boldsymbol{u}_{id} \\ &=\begin{bmatrix}\frac{-L_i}{L_{hi}}\cos\beta_i & \frac{1}{L_{hi}}\sin\beta_i\\ L_i\sin\beta_i & \cos\beta_i\end{bmatrix}\boldsymbol{u}_{id}\end{aligned} \tag{21}$$

is always well determined for the off-axle hitching. In case c2) one cannot utilize (21) due to singularity of matrix $\boldsymbol{J}_i(\beta_i)$. Following work [12] one proposes the alternative transformation (being in fact a control function)

$$\boldsymbol{u}_{i-1d}=\boldsymbol{\Psi}_i^{\text{on}}(\boldsymbol{u}_{id},\beta_i,\dot{\beta}_{id}), \tag{22}$$

with mapping

$$\boldsymbol{\Psi}_i^{\text{on}}(\boldsymbol{u}_{id},\beta_i,\dot{\beta}_{id})\triangleq\begin{bmatrix}k_i(\beta_{id}-\beta_i)+\omega_{id}+\dot{\beta}_{id}\\ \zeta\,|L_i\sin\beta_i\omega_{id}+\cos\beta_i v_{id}|\end{bmatrix}, \tag{23}$$

where $k_i>0$ and $\zeta\in\{-1,+1\}$ are the design parameters, whereas

$$\beta_{id}(\boldsymbol{u}_{id})\triangleq\text{Atan2c}\,(\zeta L_i\omega_{id},\zeta v_{id})\in\mathbb{R}, \tag{24}$$

and $\dot{\beta}_{id}$ is a feed-forward term resulting from time-differentiation of formula (24). The bi-valued factor ζ helps one confine evolution of angle (24) to the appropriate quadrants and avoid in this way the so-called jackknife phenomenon in vehicle joints, see [11], [12]. Now, according to the type of hitching one may iteratively apply transformations (20) or (22) for $i=1,\dots,N$ to obtain the resultant transformation which maps desired velocity (18) to the desired tractor-body velocity

$$\boldsymbol{u}_{0d}\triangleq\boldsymbol{\Psi}_1\circ\ldots\circ\boldsymbol{\Psi}_N, \tag{25}$$

where for $i=1,\dots,N$

$$\boldsymbol{\Psi}_i:=\begin{cases}\boldsymbol{\Psi}_i^{\text{off}}\text{ defined by (21)} & \text{if}\quad L_{hi}\neq0,\\ \boldsymbol{\Psi}_i^{\text{on}}\text{ defined by (23)} & \text{if}\quad L_{hi}=0.\end{cases} \tag{26}$$

As a consequence, equation (25) determines desired instantaneous velocities for the tractor body which allow forcing control function $\boldsymbol{\Phi}(\boldsymbol{e})$ on virtual input $\boldsymbol{u}_N$ of the guidance segment.

Remark 1: Worth noting that (25) with (11) determines in fact a cascaded interconnection of the outer-loop VFO control function $\boldsymbol{\Phi}(\boldsymbol{e})$ – with feedback from output error (6) – and the inner-loop velocity transformation being a product of mappings (21) and/or (23) according to the types of joints present in a vehicle. Detailed explanation of particular control components used in the cascaded VFO control law can be found in our prior papers [9], [12], [28].

Having determined resultant transformation (25) we propose to define suggested tractor-body velocities (8) as follows:

$$\boldsymbol{u}_{0s}(t)=\begin{bmatrix}\omega_{0s}(t)\\ v_{0s}(t)\end{bmatrix}\triangleq\begin{cases}\boldsymbol{u}_{0d}(t) & \text{for}\quad\|\boldsymbol{e}_w\|>\delta\\ \boldsymbol{0}_{2\times1} & \text{for}\quad\|\boldsymbol{e}_w\|\leq\delta\end{cases} \tag{27}$$

where $\delta\geq0$ represents the docking accuracy prescribed by the designer, and $\boldsymbol{e}_w$ is the weighted error defined by (7).

Remark 2: Let us briefly explain in what extent one may expect that suggested velocities (27) with definitions (25) and (11)-(17) are effective in achieving control objectives O1 and O2 stated in Section II-C. The answer results from our prior works which addressed direct application of control law (27) into the N-trailer robotic vehicles. In particular, achievement of objectives O1 and O2 with usage of (27) for the I-T version of the VFO control law was formally proven and verified by simulations in [12] for the robots equipped with on-axle hitching. Control law (27) with the F-T version of the VFO controller was experimentally validated in [12] with a standard three-trailer robotic vehicle. Achievement of objectives O1 and O2 for nSNT robots under assumption A1 was formally addressed and numerically verified in [9], while the experimental results obtained with a nS3T robotic vehicle were reported in [28]. Worth noting that avoiding the jackknife phenomenon in the nSNT and GNT vehicles controlled by law (27) requires backward motion strategy ($\sigma=-1$) if $L_{hi}>0$ or forward motion strategy ($\sigma=+1$) if $L_{hi}<0$ (see [9]).

B. The form of control suggestions for a human operator

Velocities (27) cannot be directly forced by a human operator because ω_0 and v_0 are not the inputs of the car-like tractor. To obtain more appropriate control suggestions for a driver, let us define the suggested motion curvature for the tractor segment

$$\kappa_{0s}(\boldsymbol{u}_{0s}(t))=\frac{\omega_{0s}(t)}{v_{0s}(t)}\in\mathbb{R}, \tag{28}$$

which is well determined for any bounded non-zero velocity $\boldsymbol{u}_{0s}(t)$ resulting from (27). Curvature (28) is a key quantity which should be reproduced accurately enough to guarantee execution of the suggested motion geometry for the N-trailer. According to equations (2) one finds a relation joining the motion curvature of a tractor body and the steering angle:

$$\kappa_0 = \frac{\omega_0}{v_0} \stackrel{(2)}{=} \frac{1}{L_0} \tan \beta_0. \quad (29)$$

Substitution of (28) to the left-hand side of (29) leads to a definition of the suggested steering angle for a car-like tractor

$$\beta_{0s} \triangleq \begin{cases} \text{Atan2}\,(v_{\text{F0}} L_0 \omega_{0s}, v_{\text{F0}} v_{0s}) & \text{for} \quad \|\boldsymbol{u}_{0s}\| > 0 \\ 0 & \text{for} \quad \|\boldsymbol{u}_{0s}\| = 0 \end{cases} \quad (30)$$

where $\text{Atan2}\,(\cdot,\cdot) : \mathbb{R} \times \mathbb{R} \mapsto (-\pi, \pi]$, while a sign of longitudinal velocity v_{F0} applied by a human driver determines proper quadrants for the suggested steering angle. According to equations (2) and (29) it is clear that the motion curvature of a car-like tractor depends only on steering angle β_0. Thus, in the case of docking maneuvers an absolute value of longitudinal velocity v_{F0} has a secondary meaning because on a kinematic level it essentially determines only a rate of maneuvers. As a consequence, any velocity profile $v_{\text{F0}}(t)$ will not be suggested to a human operator, and selection of $|v_{\text{F0}}(t)|$ will be fully left at a human driver disposal. Worth noting here, that in practice a human driver should force velocity $|v_{\text{F0}}(t)|$ with care taking into account such issues like vehicle dynamics (neglected by the assistant), a level of his/her driving skills and safety of the maneuvers[3]. Hereafter we assume that v_{F0} can be almost freely commanded by a pedal (it may be even time varying) however its sign should be kept constant, that is a human operator during maneuvers can employ only the non-positive or non-negative velocity of a front wheel:

$$v_{\text{F0}}(t) : \; v_{\text{F0}}(t_1) v_{\text{F0}}(t_2) \geq 0, \quad \forall t_1, t_2 \in [0, \infty). \quad (31)$$

Finally, by introducing the steering error

$$e_\beta(t) \triangleq \beta_{0s}(t) - \beta_0(t), \quad (32)$$

one can state that achievement of control objectives O1 and O2 formulated in Section II-C can be ensured by enforcing convergence of the steering error (32) to zero.

The proposed control-assistance system has been illustrated by the scheme in Fig. 4. From the control-theoretic viewpoint the system has a multi-loop cascaded structure. Functionally, it consists of three essential subsystems: the control assistant (fully automated and represented by the control law (27) with definitions (25) and (11)), the human-control subsystem represented by a human driver equipped with a standard steering wheel and a speed pedal, and the interface subsystem which enables an interconnection between the assistant and the human operator. According to the scheme, a human driver works as a steering controller (using the feedback from angle β_0) and as a commander of longitudinal velocity (31). In practice, angular velocity ω_{F0} cannot be directly forced by a human driver because steering is a dynamical process influenced by the inertia and friction of the steering mechanism. Steering dynamics can be approximated by a simple linear model $I\dot{\omega}_{\text{F0}} + b\omega_{\text{F0}} = m_H$, where I and b are, respectively, the effective inertia and effective damping coefficient of the entire steering mechanism (perceptible on the driver side), whereas m_H denotes a torque directly exerted by a human operator on the steering wheel. A human driver, as a steering controller, is a part of the closed-loop system presented in Fig. 5, where $f_H(\beta_{0s}, \beta_0, t)$ represents a reaction of a human driver to current values of angles β_{0s} and β_0 by applying some feedback control strategy which minimizes steering error $|e_\beta|$. Explicit form of strategy f_H is usually unknown in practice and depends on various factors like skills, experience, and psychophysical state of a driver. However, if the driver reaction $f_H(\beta_{0s}, \beta_0, t)$ is effective enough with respect to the dynamics of a steering mechanism[4] one can neglect small transient-state effects and virtually treat velocity ω_{F0} as a control signal forced by the driver and applied as a steering input to the N-trailer kinematics (see Fig. 5). A human operator is also responsible for supervision of the maneuvering process, which is supported by information on the current vehicle configuration $\boldsymbol{q}$ provided through the HMI (see the supervision loop in Fig. 4). Worth stressing that the proposed passive control-assistance system not only does not exclude a human factor from the loop, but it still leaves the responsibility for safe maneuvers and control decisions on the driver side.

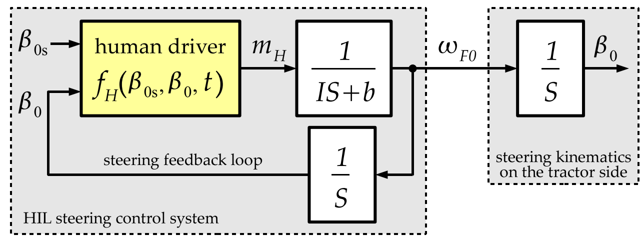

Fig. 5. Block scheme of the HIL (Human In the Loop) steering control system where a human driver plays a role of steering controller $f_H(\beta_{0s}, \beta_0, t)$, while velocity ω_{F0} is treated as a steering control input to the N-trailer kinematics (S denotes the complex operator).

IV. Laboratory-scale experiments

A. Brief comments on the experimental testbed

The experiments have been conducted by utilization of the 3-trailer articulated vehicle shown in Fig. 6. Thanks to the adjustable hitching offsets the vehicle admits selection of various kinematic structures – in particular nSNT and GNT for $N \leq 3$. The control-assistant subsystem was implemented on the vehicle board using the DSP floating-point processor TMS320F28335, and worked with sampling frequency of $100\,$Hz. The suggested tractor-body velocities (27) were sent by a wireless link to a remote human-operator console (located on a mobile computer outside the vehicle) equipped with a graphical HMI. Suggested steering angle (30) was computed by the console based on the received velocities (27) and

[3] It is recommended to select values of $|v_{\text{F0}}(t)|$ sufficiently small to keep variability of $\beta_{0s}(t)$ on the acceptable level (especially for vehicles with a large number of trailers) and to avoid substantial excitation of vehicle dynamics.

[4] Often supported by the power steering gear provided in a vehicle.

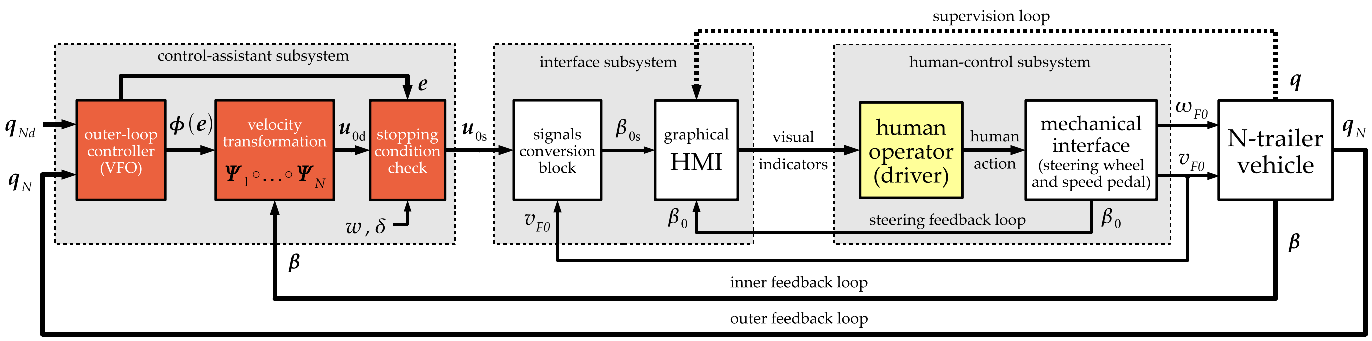

Fig. 4. Functional scheme of the passive control-assistance system for N-trailers with utilization of the cascaded VFO control law in a role of the assistant.

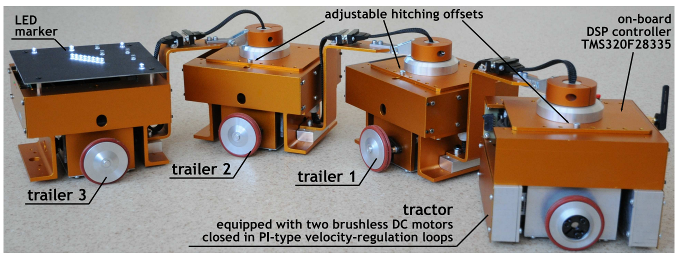

Fig. 6. The experimental 3-trailer vehicle with adjustable hitching offsets.

longitudinal velocity v_{F0} commanded by a driver. Posture $\boldsymbol{q}_N$ of the guidance segment was estimated on-line by the use of a simple sensory fusion mechanism in a form of the linear weighted combination

$$\hat{\boldsymbol{q}}_N = w_1 \hat{\boldsymbol{q}}_{Np} + w_2 \hat{\boldsymbol{q}}_{Nv}, \quad w_1 + w_2 = 1, \tag{33}$$

where $\hat{\boldsymbol{q}}_{Nv}$ denotes the estimate computed with sampling time $T_v = 25\,\text{ms}$ by an external vision system upon observation of a LED marker mounted on the last trailer, while $\hat{\boldsymbol{q}}_{Np}$ is the posture predicted upon the software kinematic model of a vehicle (see [28] for more details). Joint angles were measured by the 14-bit absolute encoders. Since a tractor of the experimental vehicle was a two-wheeled differentially-driven cart only mimicking the car-like kinematics, the command $v_{F0}(t)$ and current steering-wheel angle $\beta_0(t)$ were used in (2) to get back the tractor-body velocities $\omega_0(t)$, $v_0(t)$ and apply them as tractor control inputs. A graphical HMI was implemented on a mobile PC computer and was provided for a human driver by the remote operator console. A human driver was equipped with a mechanical interface in the form of the Logitech steering wheel and a velocity pedal. Additional implementation details can be found in [11].

B. Description of the HMI

A view of the graphical HMI has been presented in Fig. 7. The main part of the interface is the two-bar indicator which shows the currently suggested (computed) steering angle (30) through the upper bar and the current steering angle β_0 by the bottom bar. A human operator can compare indications on-line to make appropriate corrections of manual steering (steering error (32) corresponds to a difference of bars positions, see Fig. 7). Additionally, the bottom bar highlights in red when the absolute value of steering error (32) exceeds a prescribed threshold (selected by the HMI menu). Due to the limited human perception, this part of the HMI has been purposefully designed in a *minimalistic* fashion in order to limit the attention level a human must pay for an interaction with the assistant. Simplification of the HMI was also possible thanks to the intentional exclusion of longitudinal velocity $v_{F0}(t)$ from the control suggestions provided for a driver (see Section III-B). To help the operator supervise a vehicle configuration, visualization of a current vehicle chain has been provided below the bar indicators. A view from the rear on-board wireless camera allows the operator to monitor safety of the maneuvers with respect to the docking area visible on the screen.

C. Selected results of backward docking

The original results presented in paper [11] illustrated how the control performance depends on a number of trailers present in a vehicle chain. In this paper we provide selected results obtained solely for the three-trailer vehicle ($N = 3$) to show effectiveness of the method for various docking scenarios under the same vehicle complexity, and also to show applicability of the concept to a vehicle equipped with mixed types of hitching not considered in [11].

The results of three experiments, denoted as A, B, and C, are presented for the practically meaningful tasks of shifted-parallel docking, U-turn docking, and perpendicular docking, respectively. Two types of vehicle structures have been selected, namely: the nS3T kinematics with positive hitching offsets for experiments A and B, and the G3T kinematics with two off-axle and single on-axle hitching for experiment C. In the considered cases transformation (25) takes the following forms:

- for nS3T kinematics

$$\begin{aligned}\boldsymbol{u}_{0d} &= \boldsymbol{\Psi}_1^{\text{off}} \circ \boldsymbol{\Psi}_2^{\text{off}} \circ \boldsymbol{\Psi}_3^{\text{off}} \\ &= \boldsymbol{J}_1^{-1}(\beta_1)\boldsymbol{J}_2^{-1}(\beta_2)\boldsymbol{J}_3^{-1}(\beta_3)\boldsymbol{u}_{3d},\end{aligned}$$

- for G3T kinematics

$$\begin{aligned}\boldsymbol{u}_{0d} &= \boldsymbol{\Psi}_1^{\text{off}} \circ \boldsymbol{\Psi}_2^{\text{off}} \circ \boldsymbol{\Psi}_3^{\text{on}} \\ &= \boldsymbol{J}_1^{-1}(\beta_1)\boldsymbol{J}_2^{-1}(\beta_2)\boldsymbol{\Psi}_3^{\text{on}}(\boldsymbol{u}_{3d}, \beta_3, \dot{\beta}_{3d}),\end{aligned}$$

with $\boldsymbol{u}_{3d} \triangleq \boldsymbol{\Phi}(\boldsymbol{e})$ in both cases (cf. (18)).

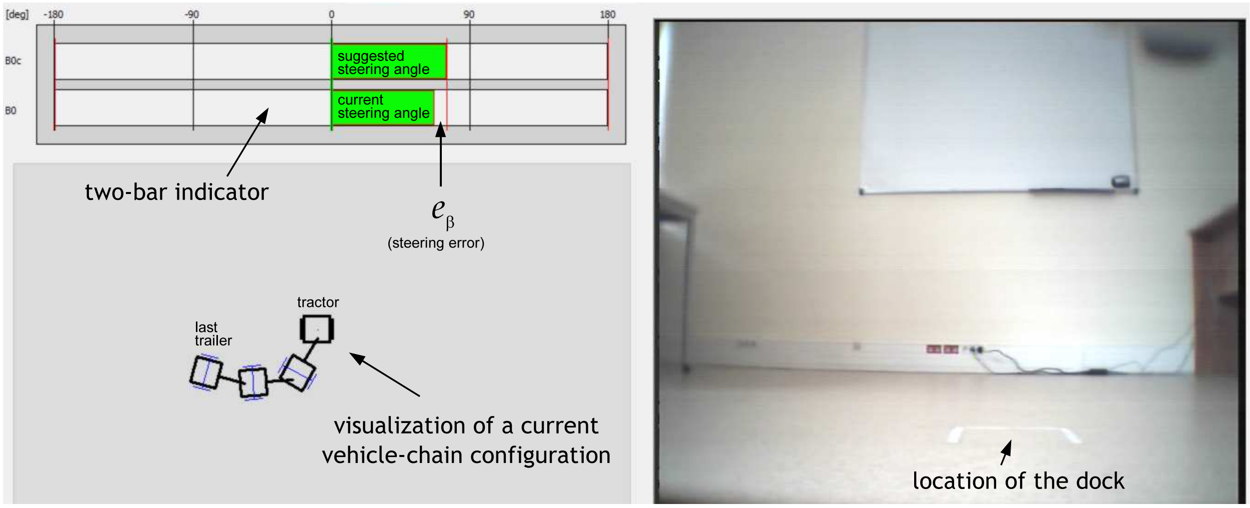

Fig. 7. Graphical Human-Machine Interface (HMI) implemented on the laboratory testbed; the view of the docking area seen from the on-board rear camera mounted on the last trailer is provided on the right side.

During all the experiments the F-T version of the VFO control law was applied (see (12)), and only constant negative velocity v_{F0} was being commanded by a driver (backward motion strategy). The experiments have been conducted for the reference posture $\boldsymbol{q}_{Nd} = \mathbf{0}$, and using the following common parameters: $L_i = 0.229\,\text{m}$, $L_{hi} = 0.048\,\text{m}$ for $i = 1, 2, 3$ (with exception of $L_{h3} = 0$ for case C[5]), $k_a = 2$, $k_p = 1$, $\eta = 0.7$, $\delta = 0.02$, $w = 0.001$, $L_0 = 0.17\,\text{m}$, $\gamma = 0.4$, and $w_1 = 0.98$, $w_2 = 0.02$ (to improve terminal attenuation of measurement noises the weights were switched into $w_1 = 1$ and $w_2 = 0$ inside the prescribed vicinity of $0.08\,\text{m}$ around the reference position). In case C, the inner-loop gain $k_3 = 20$ has been selected for transformation $\boldsymbol{\Psi}_3^{\text{on}}$ with the bi-valued factor taken as $\zeta \equiv \sigma$, while the feed-forward term $\dot{\beta}_{3d}$ has been omitted to simplify an implementation[6].

The results of three manual backward docking maneuvers supported with the proposed control-assistance system have been presented in Fig. 8. Analyzing the plots one may find quite smooth and non-oscillatory motion of the guiding segment in all three successfully accomplished trials with the characteristic directing effect, especially beneficial in the docking task. Worth emphasizing that the approaching phase to the dock did not require application of any motion planning for the guidance segment. Both the directing effect and the non-oscillatory (non-zigzag) movement of the guidance segment result solely from the characteristic properties of the VFO control law (11) applied in the control assistant (see [9], [14]). As a consequence, the obtained *natural* docking maneuvers allow avoiding potential collisions with the a priori known dock boundaries without exploiting any direct boundaries observation nor any specialized collision avoidance strategy. More oscillatory terminal behavior of the suggested steering angle β_{0s} was caused by the noise-sensitivity of the VFO control strategy increasing in a small neighborhood of the reference position (similar phenomenon occurs in practical maneuvering performed by professional drivers). Worth noting that all the tests were performed by the operator unexperienced in professional tractor-trailer maneuvers. Successful completion of the tasks turned out to be virtually impossible without a help of the proposed control-assistance system.

[5]G3T kinematics selected in case C corresponds to the vehicle comprising a tractor with a single-axle trailer followed by the so-called *full-trailer* (i.e. serial connection of a dolly with a semitrailer).

[6]In [12] it was shown that for sufficiently high gains k_i omission of the feed-forward term in (23) still allows preserving acceptable control performance.

V. Conclusions

The results presented in the paper indicate that the passive control-assistance system can be efficiently utilized to help human drivers accomplish precise docking maneuvers with N-trailer vehicles. High scalability and modularity of the cascaded VFO control law, which is the core of the assistant block, let one easily apply the method to N-trailers with various kinematics and with various number of trailers. From a practical perspective, the assistance system can be helpful not only for unexperienced drivers who do not possess appropriate skills involved by the task, but also for professional drivers making their everyday work easier. Thanks to passivity of the proposed solution, it can be more easily applicable to commercial vehicles than the active control-assistance systems proposed in the literature.

However, some issues still remain to be solved. First, reliable measurements of the vehicle configuration variables is an engineering challenge in the field applications. Second, the control-assistant algorithm could be extended with ability of the obstacles collision avoidance when docking maneuvers must be performed in a highly cluttered environment. Finally, the HMI interface may require further development in order to maximally reduce a level of human perception paid for interaction with the assistant subsystem. In this context, it seems promising for example to superimpose the supervisory-camera view with the bar indicators to narrow the required view-angle of a driver when interacting with the HMI. In the authors' opinion, all the mentioned issues should be

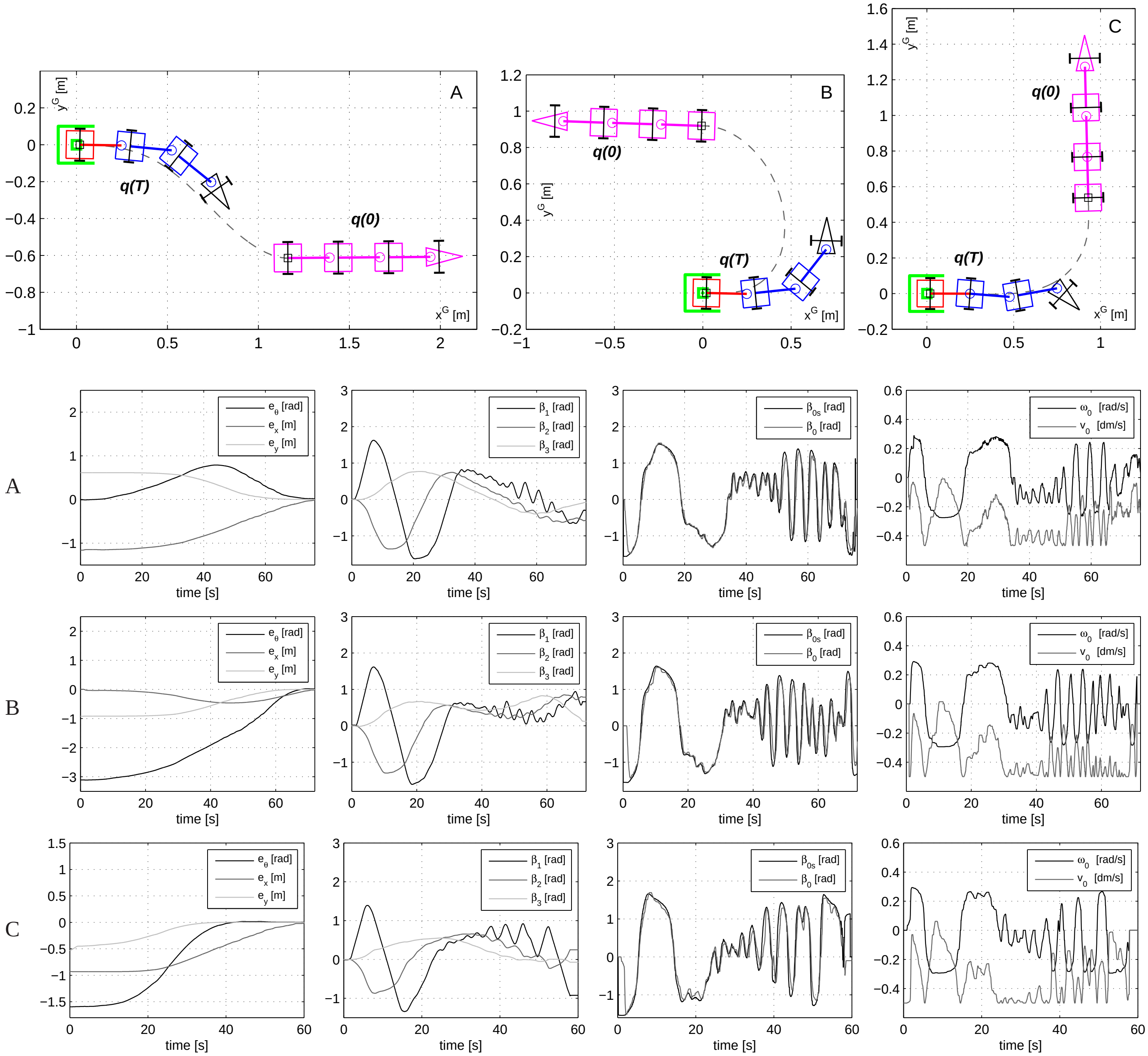

Fig. 8. Experimental results of three backward docking maneuvers performed with the help of the passive control-assistance system: (A) shifted-parallel docking with nS3T vehicle, (B) U-turn docking with nS3T vehicle, and (C) perpendicular docking with G3T vehicle (the last hitching is of on-axle type); initial vehicle configuration $\boldsymbol{q}(0)$ has been highlighted in magenta, the guidance segment has been highlighted in red, while the reference dock highlighted in green has been located at point $\boldsymbol{q}_{Nd} = \mathbf{0}$. The commanded tractor-body velocities, ω_0 and v_0, have been presented in the last column.

carefully addressed, at least in some extent, when commercial application of the system is considered.

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Maciej M. Michałek received the M.Sc. degree and the Ph.D. degree in Automation and Robotics from Poznan University of Technology (PUT), Poland, in 2001 and 2006, respectively. He is currently an Assistant Professor with Chair of Control and Systems Engineering at Faculty of Computing of PUT. His research focuses on control design problems for nonholonomic systems, especially for mobile robots and N-trailer vehicles, and for systems with non-minimum-phase dynamics. He is a member of IEEE (M'09), Robotics & Automation Society, and Control Systems Society. Since 2011, he is an Editorial Board member of *Journal of Intelligent & Robotic Systems*.

Marcin Kiełczewski received the M.Sc. degree and the Ph.D. degree in Automation and Robotics from Poznan University of Technology (PUT), Poland, in 2000 and 2010, respectively. He is currently an Assistant Professor with Chair of Control and Systems Engineering at Faculty of Computing of PUT where he teaches and conducts research. His research interests include image processing and applications of vision systems in robotics and automation.