Unified approach to trajectory tracking and set-point control for a front-axle driven car-like mobile robot

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Outline

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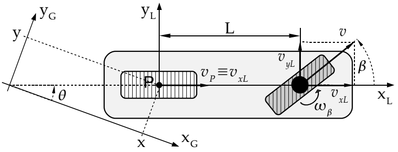
Motivation

For the car-like kinematics we are looking for:

- A unified control law which allows solving both tracking and regulation control tasks
- Intuitive control strategy with interpretable control components
- Solution not requiring any state transformation
- Resultant closed-loop system with predictable, non-oscillatory, and fast transients
- Controller which is easily tunable

Proposition: cascaded solution using the Vector-Field-Orientation (VFO) approach

Front-Driven (FD) car-like mobile robot



$L > 0$ – wheel base

P – guidance point

FD robot kinematics

$$\begin{bmatrix} \dot{\beta} \\ \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ \frac{1}{L} \sin \beta \\ \cos \beta \cos \theta \\ \cos \beta \sin \theta \end{bmatrix} u_2 \qquad (1)$$

Configuration vector

$$\boldsymbol{q} = [\beta \ \theta \ x \ y]^T \in [-\beta_m, \beta_m] \times \mathbb{R} \times \mathbb{R}^2 \qquad (2)$$

$\beta_m < \frac{\pi}{2}$: constrained motion curvature
$\beta_m = \infty$: unlimited motion curvature ($\beta \in \mathbb{R}$)
$\beta_m = \frac{\pi}{2}$: unlimited mot. curvature but simpler steering mechanism

Control input

$$\boldsymbol{u} = [u_1 \ u_2]^T \triangleq [\omega_\beta \ v] \in \mathbb{R}^2 \qquad (3)$$

Control problem – assumptions

Assumptions

A1. The reference $\boldsymbol{q}_t = [\beta_t \ \theta_t \ x_t \ y_t]^T \in [-\beta_m, \beta_m] \times \mathbb{R} \times \mathbb{R}^2$:

- $\boldsymbol{q}_t := [\beta_t(\tau) \ \theta_t(\tau) \ x_t(\tau) \ y_t(\tau)]^T = \boldsymbol{q}_t(\tau)$ for the tracking task
- $\boldsymbol{q}_t(\tau)$ is a solution of $\dot{\boldsymbol{q}}_t(\tau) = \boldsymbol{g}_1 u_{1t}(\tau) + \boldsymbol{g}_2(\boldsymbol{q}_t(\tau)) u_{2t}(\tau)$
- $\boldsymbol{q}_t(\tau)$ is sufficiently smooth such that: $\dot{u}_{1t}(\tau), \dot{u}_{2t}(\tau), \ddot{u}_{2t}(\tau) \in \mathcal{L}_\infty$
- $\boldsymbol{q}_t(\tau)$ is persistently exciting: $\forall \tau \geqslant 0 \ u_{2t}(\tau) \cos \beta_t(\tau) \neq 0$
- $\boldsymbol{q}_t := [0 \ \theta_t \ x_t \ y_t]^T$ for the regulation task

A2. all components of configuration $\boldsymbol{q}$ are measurable

A3. parameter value L is perfectly known

Control problem – definition

Problem definition

Given a reference $\boldsymbol{q}_t$, determine a feedback control law $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{q}_t, \boldsymbol{q}, \cdot)$ for FD car-like kinematics, which guarantees convergence of the configuration error

$$\boldsymbol{e}(\tau) = \begin{bmatrix} e_\beta(\tau) \\ e_\theta(\tau) \\ e_x(\tau) \\ e_y(\tau) \end{bmatrix} = \begin{bmatrix} e_\beta(\tau) \\ \overline{\boldsymbol{e}}(\tau) \end{bmatrix} \triangleq \begin{bmatrix} \beta_t - \beta(\tau) \\ f_\theta(\theta_t - \theta(\tau)) \\ x_t - x(\tau) \\ y_t - y(\tau) \end{bmatrix} \quad (4)$$

in the sense that

$$\lim_{\tau \to \infty} \|\boldsymbol{e}(\tau)\| \leqslant \epsilon, \qquad \epsilon \geqslant 0, \quad (5)$$

where: $f_\theta(\cdot) : \mathbb{R} \mapsto \mathbb{S}^1$, and ϵ is some vicinity of the origin.

$\epsilon = 0 \Rightarrow$ asymptotic convergence
$\epsilon > 0 \Rightarrow$ practical convergence (ultimate boundedness)

Outline

Control design principle

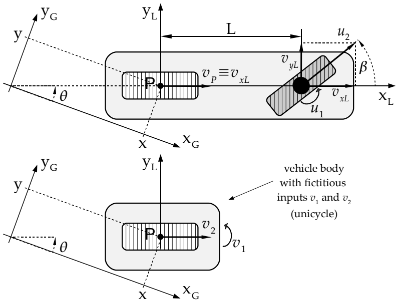
Main concept

Design the feedback control law for the FD car-like kinematics using the unicycle feedback controller applied to the vehicle body subsystem.

Three main design steps:

1. Reformulation of control inputs and decomposition of the car-like kinematics
2. Application of a unicycle feedback law to the vehicle-body subsystem (VFO method)
3. Recovering original inputs of the car-like kinematics

Step1: reformulation of the FD car-like kinematics



Fictitious inputs of the vehicle body

$$u_2(1/L)\sin\beta =: v_1 \tag{6}$$

$$u_2\cos\beta =: v_2 \tag{7}$$

v_1 – angular velocity of the vehicle body

v_2 – longitudinal velocity of point P

Decomposed FD kinematics

$$\dot{\beta} = u_1, \tag{8}$$

$$\begin{bmatrix}\dot{\theta}\\ \dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix} v_1 + \begin{bmatrix}0\\ \cos\theta\\ \sin\theta\end{bmatrix} v_2. \tag{9}$$

(9) is the vehicle-body unicycle-like subsystem (!)

Vehicle configuration decomposed into:
body configuration $\overline{\boldsymbol{q}} = [\theta\ x\ y]^T$ and steering wheel angle β (internal variable)

Step2: fictitious inputs defined as feedback functions for vehicle body

To guarantee that the vehicle body configuration $\overline{\boldsymbol{q}}$ will converge to the reference $\overline{\boldsymbol{q}}_t$ we define:

$$v_1 \triangleq \Phi_1(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot), \qquad v_2 \triangleq \Phi_2(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot), \tag{10}$$

where $\Phi_1(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot)$ and $\Phi_2(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot)$ are the differentiable feedback control functions for the vehicle body subsystem, which applied for the unicycle-like kinematics

$$\begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Phi_1(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot) + \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} \Phi_2(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot)$$

ensure that

$$\lim_{\tau \to \infty} \| \overline{\boldsymbol{q}}_t(\tau) - \overline{\boldsymbol{q}}(\tau) \| = 0.$$

We propose to design $\Phi_1(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot)$ and $\Phi_2(\overline{\boldsymbol{q}}_t, \overline{\boldsymbol{q}}, \cdot)$ according to the VFO control strategy.

Step3a: recovering the original input u_2 and the steering angle formula

After substitution $v_1 := \Phi_1$ and $v_2 := \Phi_2$ one obtains:

$$u_2(1/L)\sin\beta = \Phi_1 \quad (11)$$

$$u_2 \cos\beta = \Phi_2 \quad (12)$$

$$\Downarrow$$

$$u_2 = \Phi_2 \cos\beta + L\Phi_1 \sin\beta \quad (13)$$

$$\beta = \arctan\left(\frac{L\Phi_1}{\Phi_2}\right) \quad \text{for} \quad \sqrt{\Phi_1^2 + \Phi_2^2} \neq 0 \quad (14)$$

Since (14) cannot be met instantaneously we introduce the *auxiliary steering variable*

$$\beta_a \triangleq \arctan\left(\frac{L\Phi_1}{\Phi_2}\right) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \text{for} \quad \sqrt{\Phi_1^2 + \Phi_2^2} \neq 0 \quad (15)$$

and the *auxiliary steering error*

$$e_{\beta a} \triangleq \beta_a - \beta \quad (16)$$

Now, to satisfy (14) it suffices to make $e_{\beta a}$ converge to zero.

Step3b: definition for the original input u_1

Since the steering dynamics are $\dot{\beta} = u_1$ let us define the steering inputs as follows:

$$u_1 \triangleq k_\beta \, e_{\beta a} + \dot{\beta}_a, \qquad k_\beta > 0, \tag{17}$$

where k_β is a design parameter, and

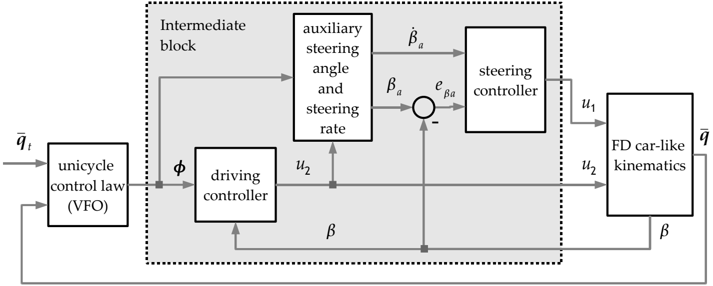
$$\dot{\beta}_a = \frac{L(\dot{\Phi}_1 \Phi_2 - \Phi_1 \dot{\Phi}_2)}{L^2 \Phi_1^2 + \Phi_2^2} \quad \text{for} \quad \sqrt{\Phi_1^2 + \Phi_2^2} \neq 0 \tag{18}$$

is a feed-forward term.

Resultant control law for FD car-like kinematics

steering controller: $u_1 = k_\beta \, e_{\beta a} + \dot{\beta}_a$ (19)

driving controller: $u_2 = \Phi_2 \cos \beta + L\Phi_1 \sin \beta$ (20)



Particular form of control law (19)-(20) results from definitions of feedback functions Φ_1 and Φ_2 . . .

Outline

VFO controller for unicycle kinematics*

*M. Michałek, K. Kozłowski: Vector-Field-Orientation feedback control method for a differentially driven vehicle, IEEE Trans. Control Sys. Techn., 18(1), 2010

orienting control: $$\Phi_1 \triangleq k_\theta e_{\theta a} + \dot{\theta}_a \quad (21)$$

pushing control: $$\Phi_2 \triangleq h_x \cos\theta + h_y \sin\theta \quad (22)$$

For tracking task

$$h_x = k_p e_x + v_x, \quad v_x = \dot{x}_t \quad (23)$$
$$h_y = k_p e_y + v_y, \quad v_y = \dot{y}_t \quad (24)$$
$$e_{\theta a} = \theta_a - \theta \quad (25)$$
$$\theta_a = \mathrm{Atan2c}\,(\sigma \cdot h_y, \sigma \cdot h_x) \quad (26)$$
$$\dot{\theta}_a = (\dot{h}_y h_x - h_y \dot{h}_x)/(h_x^2 + h_y^2) \quad (27)$$
$$\sigma \triangleq \mathrm{sgn}(v_{2t}) = \mathrm{sgn}(u_{2t} \cos\beta_t) \quad (28)$$

For regulation task (*almost stabilizer*)

$$h_x = k_p e_x + v_x, \quad v_x = -\eta\sigma \,\|\, \overline{\boldsymbol{e}}^* \,\|\, \cos\theta_t \quad (29)$$
$$h_y = k_p e_y + v_y, \quad v_y = -\eta\sigma \,\|\, \overline{\boldsymbol{e}}^* \,\|\, \sin\theta_t \quad (30)$$
$$e_{\theta a} = \theta_a - \theta \quad (31)$$
$$\theta_a = \mathrm{Atan2c}\,(\sigma \cdot h_y, \sigma \cdot h_x) \quad (32)$$
$$\dot{\theta}_a = (\dot{h}_y h_x - h_y \dot{h}_x)/(h_x^2 + h_y^2) \quad (33)$$
$$\sigma \triangleq \mathrm{sgn}(e_{x0} \cos\theta_t + e_{y0} \sin\theta_t) \quad (34)$$

Design coefficients: $k_\theta > k_p > 0$ and $\eta \in (0, k_p)$

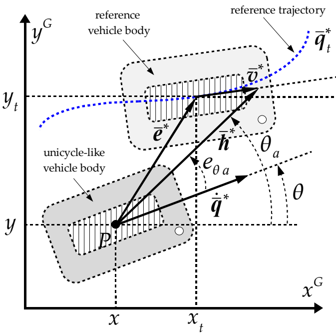
Decision factor $\sigma \in \{-1, +1\}$

$\mathrm{Atan2c}\,(\cdot,\cdot) : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}$

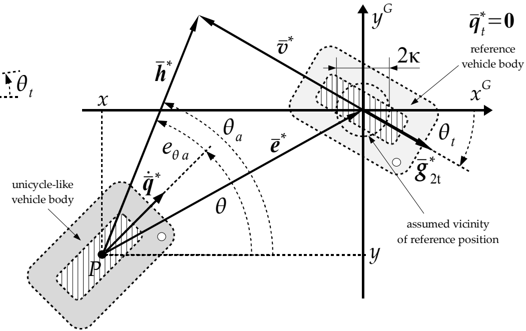
$\|\, \overline{\boldsymbol{e}}^* \,\| = \sqrt{e_x^2 + e_y^2}$

VFO control – geometrical interpretations for the unicycle

trajectory tracking $(k_p = 1)$



set-point regulation $(k_p = 1)$



$$\overline{\boldsymbol{h}}^* = [h_x \; h_y]^T, \quad \overline{\boldsymbol{e}}^* = [e_x \; e_y]^T, \quad \bar{\boldsymbol{v}}^* = [v_x \; v_y]^T, \quad \dot{\boldsymbol{q}}^* = [\dot{x} \; \dot{y}]^T = v_2[\cos\theta \;\; \sin\theta]^T$$

Outline

1. Introduction
2. General control structure
3. VFO method application
4. Simulation results
5. Remarks

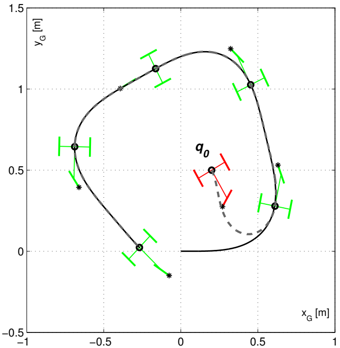
Sim1: trajectory tracking (forward motion)

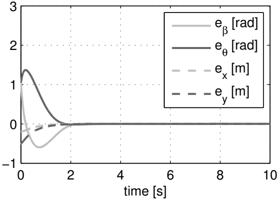
Simulation conditions

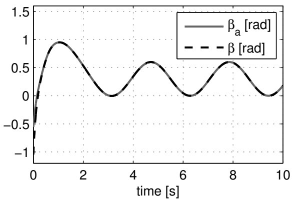
$$\boldsymbol{q}_0 = [-\tfrac{\pi}{3} \quad -\tfrac{\pi}{3} \quad 0.2 \quad 0.5]^T$$
$$\boldsymbol{q}_{t0} = [0 \quad 0 \quad 0 \quad 0]^T$$
$$u_{1t}(\tau) = 0.6\sin(2\tau)\ \text{rad/s}, \quad u_{2t} = 0.4\ \text{m/s}$$

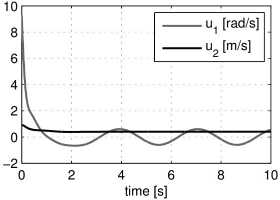
Parameter values

$$L = 0.2\,\text{m}$$
$$k_\beta = 10, \quad k_\theta = 5, \quad k_p = 2$$









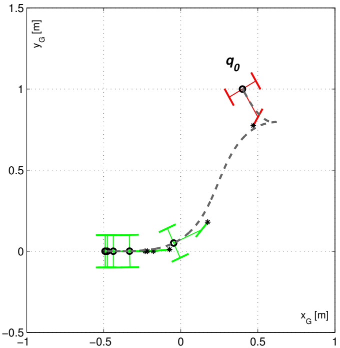
Sim2: parking maneuvers (backward motion)

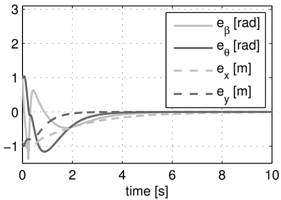
Simulation conditions

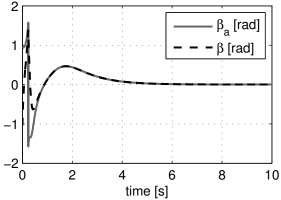
$$\boldsymbol{q}_0 = [-\tfrac{\pi}{3} \quad -\tfrac{\pi}{3} \quad 0.4 \quad 1.0]^T$$
$$\boldsymbol{q}_t = [0 \quad 0 \quad -0.5 \quad 0]^T$$

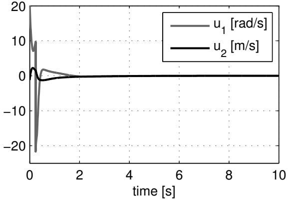
Parameter values

$$L = 0.2\,\mathrm{m}$$
$$k_\beta = 10, \quad k_\theta = 5, \quad k_p = 2, \quad \eta = 1.5$$



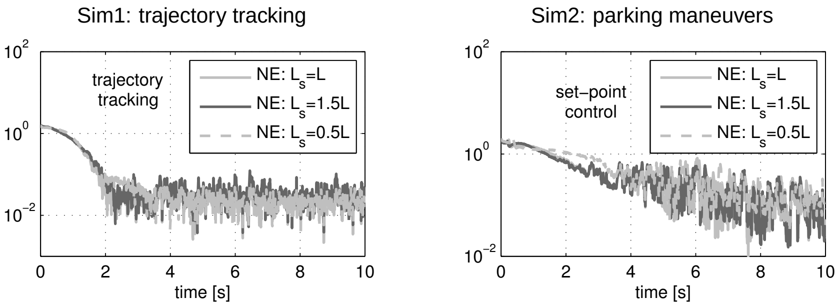






Sim3: robustness tests

Time plots of $\|\bar{e}(\tau)\|$ after applying in the controller the nominal ($L_s := L$), 50% overestimated ($L_s := 1.50L$), and 50% underestimated ($L_s := 0.5L$) robot parameter, adding simultaneously the white Gaussian measurement noises to feedback signals with standard deviation $\mathrm{Std} = 0.001$ rad



Outline

Final remarks

- Presented approach as an extension of the VFO application to FD car-like kinematics
- Trajectory tracking and set-point regulation treated in the unified manner
- General control structure allows applying alternative unicycle controllers**
- Application of the method to the rear-driven car-like kinematics – possible**

**M. Michałek, K. Kozłowski: Feedback control framework for car-like robots using the unicycle controllers, accepted for publication in Robotica, 2011

Thank you for attention

Indeterminacy of β_a and $\dot{\beta}_a$

Definitions (15) and time-derivative (18) are not determined for time instants $\underline{\tau}$ when $\mathbf{\Phi}(\underline{\tau}) = \mathbf{0}$. In this case one can introduce additional definitions, for example $\beta_a(\underline{\tau}) := \lim_{\tau \to \underline{\tau}} \beta_a(\tau)$ and $\dot{\beta}_a(\underline{\tau}) := 0$ activated for all $\underline{\tau}$ when $\| \mathbf{\Phi}(\underline{\tau})\| = 0$. In practice, one may prefer replace the last condition by $\| \mathbf{\Phi}(\underline{\tau})\| < \delta$ with $\delta > 0$ being a sufficiently small vicinity of zero.

Indeterminacy of θ_a and $\dot{\theta}_a$ (tracking case)

To cope with the indeterminacy of terms θ_a and $\dot{\theta}_a$ when $\overline{\boldsymbol{h}}^* = \mathbf{0}$ we propose to introduce additional definitions for θ_a and $\dot{\theta}_a$ in a small ε-vicinity of point $\overline{\boldsymbol{h}}^* = \mathbf{0}$ as follows:

$$\theta_a \triangleq \theta_a(\tau^-) \quad \text{and} \quad \dot{\theta}_a \triangleq 0 \qquad \text{for} \quad \left\| \overline{\boldsymbol{h}}^* \right\| < \varepsilon, \tag{35}$$

with $0 < \varepsilon < \inf_\tau |u_{2t}(\tau) \cos \beta_t(\tau)|$, and τ^- being determined by $\left\| \overline{\boldsymbol{h}}^*(\tau^-) \right\| = \varepsilon$. Note that other definitions together with the control input ones remain unchanged. Since the indeterminacy point is non-attracting and non-persistent, all the convergence results stay valid.

Indeterminacy of θ_a and $\dot{\theta}_a$ (regulation case)

In the regulation case definitions for θ_a and $\dot{\theta}_a$ are not defined for $\overline{\boldsymbol{e}}^* = \mathbf{0}$. Since the point $\overline{\boldsymbol{e}}^* = \mathbf{0}$ is reachable only at infinity, the VFO set-point controller belongs to the so-called *almost stabilizers*. Although in practice, one may need a well defined controller also at this point. Introducing definitions

$$\theta_a \triangleq \theta_a(\tau_\kappa), \quad \dot{\theta}_a \triangleq 0, \quad \beta_a = \dot{\beta}_a \triangleq 0, \quad u_2 \triangleq 0 \tag{36}$$

being active for $\tau \geqslant \tau_\kappa$ where $\|\overline{\boldsymbol{e}}^*(\tau \geqslant \tau_\kappa)\| < \kappa$ with some assumed $\kappa > 0$, leads to the ultimate boundedness of error (4) solving the control problem with $\epsilon = \epsilon(\kappa, e_\theta(\tau_\kappa)) > 0$ and with $\lim_{\tau \to \infty} e_\beta(\tau) = 0$.